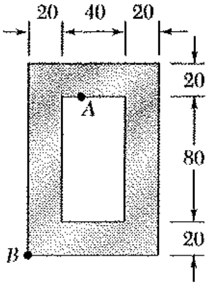
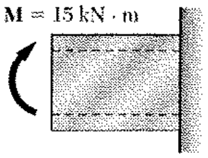
Chapter 4

Problem 4.1

4.1 and 4.2 Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.



Dimensions in mm



For rectangle $I = \frac{1}{12} b h^3$

Outside rectangle: $I_1 = \frac{1}{12}(80)(120)^3$

$I_1 = 11.52\times10^{6}\ \text{mm}^4 = 11.52\times10^{-6}\ \text{m}^4$

Cutout: $I_2 = \frac{1}{12}(40)(80)^3$

$I_2 = 1.70667\times10^{6}\ \text{mm}^4 = 1.70667\times10^{-6}\ \text{m}^4$

Section: $I = I_1 - I_2 = 9.81333\times10^{-6}\ \text{m}^4$

(a) $y_A = 40\ \text{mm} = 0.040\ \text{m}$ $\quad \sigma_A = -\frac{My_A}{I} = -\frac{(15\times10^3)(0.040)}{9.81333\times10^{-6}} = -61.6\times10^{6}\ \text{Pa}$

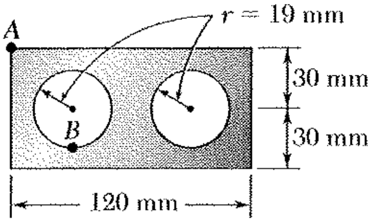
$\sigma_A = -61.6\ \text{MPa}$ ◄

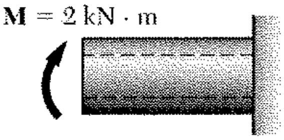
(b) $y_B = -60\ \text{mm} = -0.060\ \text{m}$ $\quad \sigma_B = -\frac{My_B}{I} = -\frac{(15\times10^3)(-0.060)}{9.81333\times10^{-6}} = 91.7\times10^{6}\ \text{Pa}$

$\sigma_B = 91.7\ \text{MPa}$ ◄

Problem 4.2

4.1 and *4.2* Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.





For the solid rectangle,

$I_1 = \frac{1}{12} b h^3 = \frac{1}{12}(0.12)(0.06)^3$

$= 2.16\times10^{-6}\ \text{m}^4$

For one circular cutout, $I_2 = \frac{\pi}{4} r^4 = \frac{\pi}{4}(0.019)^4$

$= 102.35\times10^{-9}\ \text{m}^4$

For the section, $I = I_1 - 2I_2 = 2.16\times10^{-6} - (2)(102.35\times10^{-9}) = 1.955\times10^{-6}\ \text{m}^4$

(a) $\underline{y_A = 0.03\ \text{m}}$ $\quad \sigma_A = -\frac{My_A}{I} = -\frac{(2000)(0.03)}{1.955\times10^{-6}} = -30.69\ \text{MPa}$

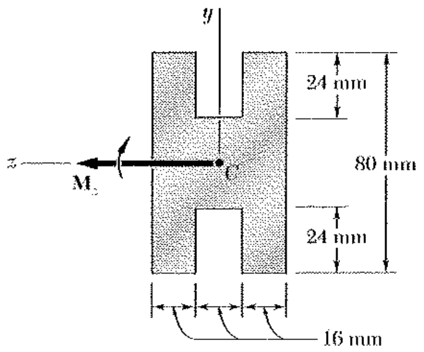
$\sigma_A = -30.7\ \text{MPa}$ ◄

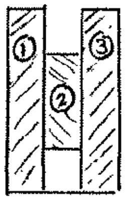
(b) $\underline{y_B = -0.019\ \text{m}}$ $\quad \sigma_B = -\frac{My_B}{I} = -\frac{(2000)(-0.019)}{1.955\times10^{-6}} = 19.44\ \text{MPa}$

$\sigma_B = 19.4\ \text{MPa}$ ◄

Problem 4.3

4.3 A beam of the cross section shown is extruded from an aluminum alloy for which $\sigma_Y = 250$ MPa and $\sigma_U = 450$ MPa. Using a factor of safety of 3.00, determine the largest couple that can be applied to the beam when it is bent about the z axis.





$$\text{Allowable stress} = \frac{\sigma_U}{F.S.} = \frac{450}{3} = 150 \text{ MPa}$$
$$= 150 \times 10^6 \text{ Pa}$$

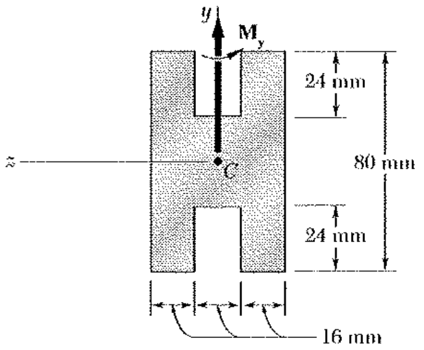
Moment of inertia about z axis.

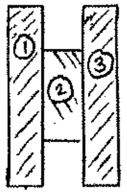
$$I_1 = \frac{1}{12}(16)(80)^3 = 682.67 \times 10^3 \text{ mm}^4$$
$$I_2 = \frac{1}{12}(16)(32)^3 = 43.69 \times 10^3 \text{ mm}^4$$
$$I_3 = I_1 = 682.67 \times 10^3 \text{ mm}^4$$
$$I = I_1 + I_2 + I_3 = 1.40902 \times 10^6 \text{ mm}^4 = 1.40902 \times 10^{-6} \text{ m}^4$$
$$\sigma = \frac{Mc}{I} \quad \text{with } c = \frac{1}{2}(80) = 40 \text{ mm} = 0.040 \text{ m}$$
$$M = \frac{I\sigma}{c} = \frac{(1.40902 \times 10^{-6})(150 \times 10^6)}{0.040} = 5.28 \times 10^3 \text{ N·m} \qquad M = 5.28 \text{ kN·m} ◄$$

Problem 4.4

4.4 Solve Prob. 4.3, assuming that the beam is bent about the y axis.

4.3 A beam of the cross section shown is extruded from an aluminum alloy for which $\sigma_Y = 250$ MPa and $\sigma_U = 450$ MPa. Using a factor of safety of 3.00, determine the largest couple that can be applied to the beam when it is bent about the z axis.





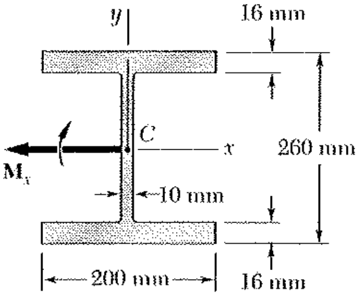
$$\text{Allowable stress} = \frac{\sigma_U}{F.S.} = \frac{450}{3.00} = 150 \text{ MPa}$$
$$= 150 \times 10^6 \text{ Pa}$$

Moment of inertia about y-axis.

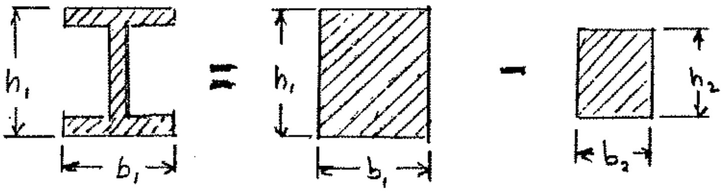
$$I_1 = \frac{1}{12}(80)(16)^3 + (80)(16)(16)^2 = 354.987 \times 10^3 \text{ mm}^4$$
$$I_2 = \frac{1}{12}(32)(16)^3 = 10.923 \times 10^3 \text{ mm}^4$$
$$I_3 = I_1 = 354.987 \times 10^3 \text{ mm}^4$$
$$I = I_1 + I_2 + I_3 = 720.9 \times 10^3 \text{ mm}^4 = 720.9 \times 10^{-9} \text{ m}^4$$
$$\sigma = \frac{Mc}{I} \quad \text{with } c = \frac{1}{2}(48) = 24 \text{ mm} = 0.024 \text{ m}$$
$$M = \frac{I\sigma}{c} = \frac{(720.9 \times 10^{-9})(150 \times 10^6)}{0.024} = 4.51 \times 10^3 \text{ N·m} \qquad M = 4.51 \text{ kN·m} ◄$$

Problem 4.5

4.5 The steel beam shown is made of a grade of steel for which $\sigma_Y = 250$ MPa and $\sigma_U = 400$ MPa. Using a factor of safety of 2.50, determine the largest couple that can be applied to the beam when it is bent about the x axis.



The moment of inertia I_x is equivalent to that of a rectangle with a cutout



Larger rectangle: $b_1 = 200$ mm $\quad h_1 = 260$ mm $\quad I_1 = \frac{1}{12} b_1 h_1^3$

$$I_1' = \frac{1}{12}(200)(260)^3 = 292.933 \times 10^6 \text{ mm}^4$$

Smaller rectangle: $b_2 = 200 - 10 = 190$ mm $\quad h_2 = 260 - (2)(16) = 228$mm

$$I_2 = \frac{1}{12}(190)(228)^3 = 187.662 \times 10^6 \text{ mm}^4$$

Section: $I_x = I_1 - I_2 = 105.271 \times 10^6 \text{ mm}^4 = 105.271 \times 10^{-6} \text{ m}^4$

$$c = \frac{260}{2} = 130 \text{ mm} = 0.130 \text{ m}$$

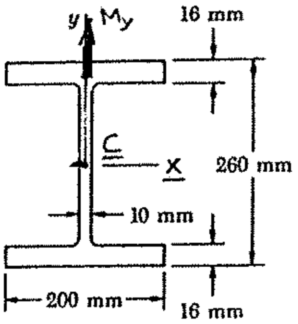
$$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{400}{2.50} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$$

$$\sigma_{all} = \frac{M_x c}{I_x} \qquad M_x = \frac{I_x \sigma_{all}}{c} = \frac{(105.271 \times 10^{-6})(160 \times 10^6)}{0.130}$$

$$= 129.564 \times 10^3 \text{ N}\cdot\text{m}$$

$M_x = 129.6$ kN·m ◄

Problem 4.6



4.6 Solve Prob. 4.5, assuming that the steel beam is bent about the *y* axis by a couple of moment M_y.

4.5 The steel beam shown is made of a grade of steel for which $\sigma_Y = 250$ MPa and $\sigma_U = 400$ MPa. Using a factor of safety of 2.50, determine the largest couple that can be applied to the beam when it is bent about the *x* axis.

For one flange, $I_f = \frac{1}{12} b_f h_f^3$

$b_f = 16$ mm $\quad h_f = 200$ mm

$I_f = \frac{1}{12}(16)(200)^3 = 10.6667 \times 10^6$ mm^4

For the web $\quad I_w = \frac{1}{12} b_w h_w^3$

$b_w = 260 - (2)(16) = 228$ mm $\quad h_w = 10$ mm

$I_w = \frac{1}{12}(228)(10)^3 = 19 \times 10^3$ mm^4

Section: $I_y = 2 I_f + I_w = 21.352 \times 10^6$ mm^4 $= 21.352 \times 10^{-6}$ m^4

$c = \frac{200}{2} = 100$ mm $= 0.100$ m

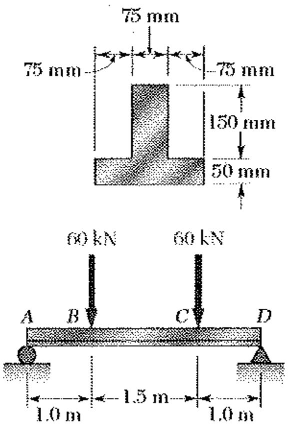
$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{400}{2.50} = 160$ MPa $= 160 \times 10^6$ Pa

$\sigma_{all} = \frac{M_y c}{I_y} \qquad M_y = \frac{I_y c}{\sigma_{all}} = \frac{(21.352 \times 10^{-6})(160 \times 10^6)}{0.100}$

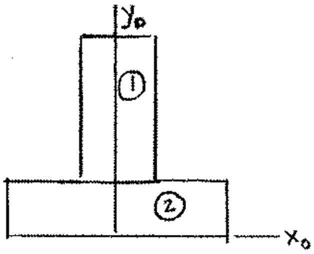
$= 34.163 \times 10^3$ N·m

$M_y = 34.2$ kN·m ◀

Problem 4.7



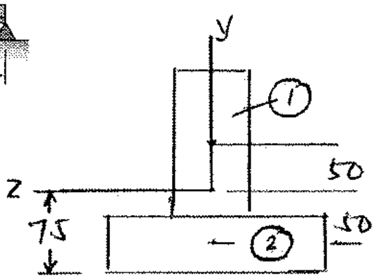
4.7 through 4.9 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.



	A	$\bar{y}_o$	$A\bar{y}_o$
①	11250	125	1406250
②	11250	25	281250
Σ	22500		1687500

$$\bar{Y}_o = \frac{1687500}{22500} = 75 \text{ mm}$$

Neutral axis lies 75mm. above the base.

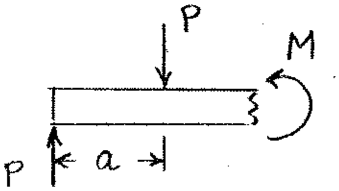


$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(0.075)(0.15)^3 + (0.01125)(0.05)^2 = 49.22\times10^{-6}\ m^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(0.225)(0.05)^3 + (0.01125)(0.05)^2 = 30.47\times10^{-6}\ m^4$$

$$I = I_1 + I_2 = 79.7\times10^{-6}\ m^4.$$

$y_{top} = 125$ mm $\quad y_{bot} = -75$ mm



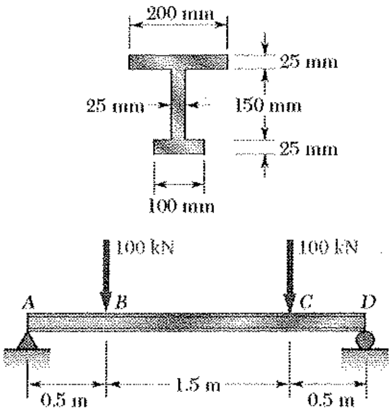
$$M - Pa = 0$$

$$M = Pa = (60)(1) = 60 \text{ kN.m.}$$

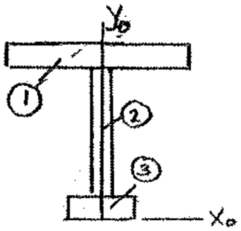
$$\sigma_{top} = -\frac{M y_{top}}{I} = -\frac{(60000)(0.125)}{79.7\times10^{-6}} = -94.1 \text{ MPa (compression)}$$

$$\sigma_{bot} = -\frac{M y_{bot}}{I} = -\frac{(60000)(-0.075)}{79.7\times10^{-6}} = 56.5 \text{ MPa (tension)}$$

Problem 4.8



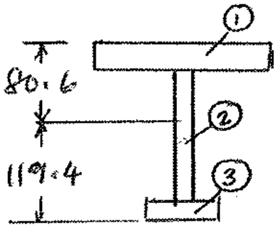
4.7 through 4.9 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.



	$(\times 10^3)$		$(\times 10^3)$
	A	$\bar{Y}_o$	$A\bar{Y}_o$
①	5	187.5	937
②	3.75	100	375
③	2.5	12.5	31.25
Σ	11.25		1343.25

$$\bar{Y}_o = \frac{1343.25}{11.25} = 119.4 \text{ mm}$$

Neutral axis lies 119.4 above the base.



$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(200)(25)^3 + (5000)(68.1)^2$$

$$= 23.448 \times 10^{6} \text{ mm}^4$$

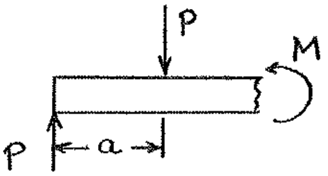
$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(25)(150)^3 + (3750)(19.4)^2$$

$$= 8.443 \times 10^{6} \text{ mm}^4.$$

$$I_3 = \frac{1}{12}b_3h_3^3 + A_3d_3^2 = \frac{1}{12}(100)(25)^3 + (2500)(106.9)^2 = 28.699 \times 10^{6} \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 60.59 \times 10^{-6} \text{ m}^4$$

$y_{top} = 80.6$ mm $\qquad y_{bot} = -119.4$ mm



$$M - Pa = 0$$

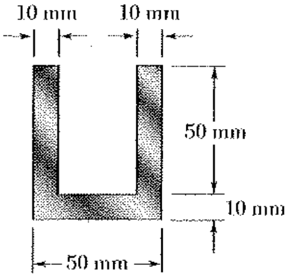
$$M = Pa = (60)(1) = 60 \text{ kN.m}$$

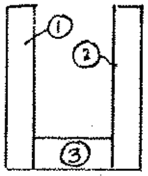
$$\sigma_{top} = -\frac{My_{top}}{I} = -\frac{(60000)(0.0806)}{60.59 \times 10^{-6}} = -79.8 \text{ MPa. (compression)}$$ ◀

$$\sigma_{bot} = -\frac{My_{bot}}{I} = -\frac{(60000)(0.1194)}{60.59 \times 10^{-6}} = 118.2 \text{ MPa (tension)}$$ ◀

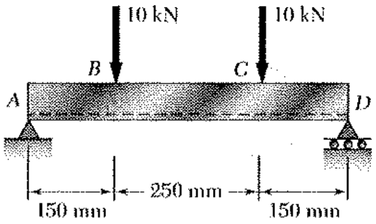
Problem 4.9

4.7 through 4.9 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.

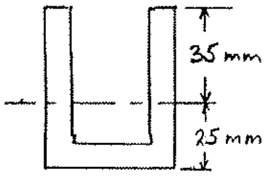




	A, mm²	$\bar{y}_o$, mm	$A\bar{y}_o$, mm³
①	600	30	18×10^3
②	600	30	18×10^3
③	300	5	1.5×10^3
	1500		37.5×10^3



$$\bar{Y}_o = \frac{37.5\times10^3}{1500} = 25 \text{ mm}$$



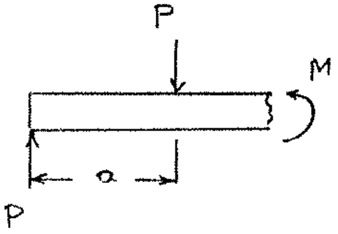
Neutral axis lies 25 mm above the base.

$$I_1 = \frac{1}{12}(10)(60)^3 + (600)(5)^2 = 195\times10^3 \text{ mm}^4 \qquad I_2 = I_1 = 195 \text{ mm}^4$$

$$I_3 = \frac{1}{12}(30)(10)^3 + (300)(20)^2 = 122.5\times10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 512.5\times10^3 \text{ mm}^4 = 512.5\times10^{-9} \text{ m}^4$$

$$y_{top} = 35 \text{ mm} = 0.035 \text{ m} \qquad y_{bot} = -25 \text{ mm} = -0.025 \text{ m}$$



$$a = 150 \text{ mm} = 0.150 \text{ m} \qquad P = 10\times10^3 \text{ N}$$

$$M = Pa = (10\times10^3)(0.150) = 1.5\times10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{top} = -\frac{My_{top}}{I} = -\frac{(1.5\times10^3)(0.035)}{512.5\times10^{-9}} = -102.4\times10^6 \text{ Pa} \qquad \sigma_{top} = -102.4 \text{ MPa}$$

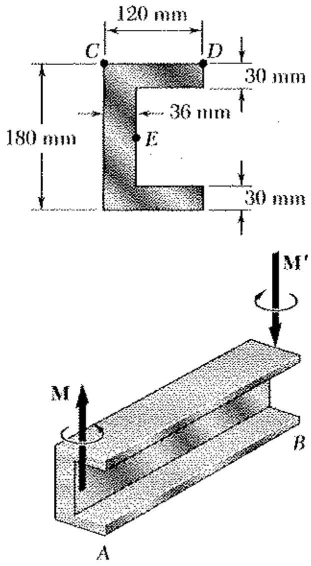
(compression) ◀

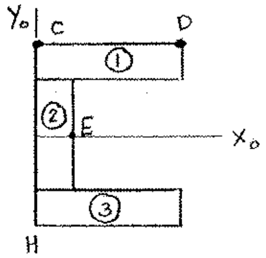
$$\sigma_{bot} = -\frac{My_{bot}}{I} = -\frac{(1.5\times10^3)(-0.025)}{512.5\times10^{-9}} = 73.2\times10^6 \text{ Pa} \qquad \sigma_{bot} = 73.2 \text{ MPa}$$

(tension) ◀

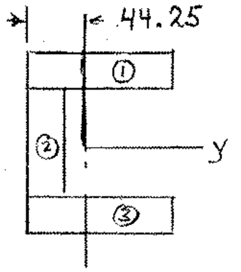
Problem 4.10

4.10 Two equal and opposite couples of magnitude $M = 25$ kN · m are applied to the channel-shaped beam AB. Observing that the couples cause the beam to bend in a horizontal plane, determine the stress at (*a*) point C, (*b*) point D, (*c*) point E.





	A, mm²	$\bar{x}_o$, mm	$A\bar{x}_o$, mm³
①	3600	60	216×10^3
②	4320	18	77.76×10^3
③	3600	60	216×10^3
Σ	11520		509.76×10^3



$$\bar{X} = \frac{509.76 \times 10^3}{11520} = 44.25 \text{ mm}$$

$$y_C = -44.25 \text{ mm} = -0.04425 \text{ m}$$

$$y_D = 120 - 44.25 = 75.75 \text{ mm} = 0.07575 \text{ m}$$

$$y_E = 36 - 44.25 = -8.25 \text{ mm} = -0.00825 \text{ m}$$

$$d_1 = 60 - 44.25 = 15.75 \text{ mm}$$
$$d_2 = 44.25 - 18 = 26.26 \text{ mm}$$
$$d_3 = d_1$$

$$I_1 = I_3 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(30)(120)^3 + (3600)(15.75)^2 = 5.2130 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(120)(36)^3 + (4320)(26.25)^2 = 3.4433 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 6.5187 \times 10^6 \text{ mm}^4 = 13.8694 \times 10^{-6} \text{ m}^4$$

$$M = 15 \times 10^3 \text{ N-m}$$

(a) Point C: $\sigma_c = -\frac{My_c}{I} = -\frac{(25 \times 10^3)(-0.04425)}{13.8694 \times 10^{-6}} = 79.8 \times 10^6$ Pa

$\sigma_c = 79.8$ MPa ◄

(b) Point D: $\sigma_D = -\frac{My_D}{I} = -\frac{(25 \times 10^3)(0.07575)}{13.8694 \times 10^{-6}} = -136.5 \times 10^6$ Pa

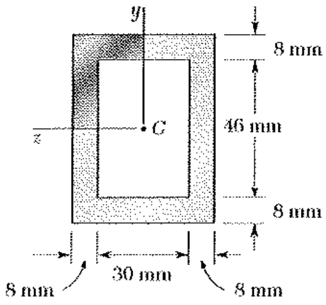
$\sigma_D = -136.5$ MPa ◄

(c) Point E: $\sigma_E = -\frac{My_E}{I} = -\frac{(25 \times 10^3)(-0.00825)}{13.8694 \times 10^{-6}} = 14.87 \times 10^6$ Pa

$\sigma_E = 14.87$ MPa ◄

Problem 4.11

4.11 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 900 N · m, determine the total force acting on the shaded portion of the beam.



The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$

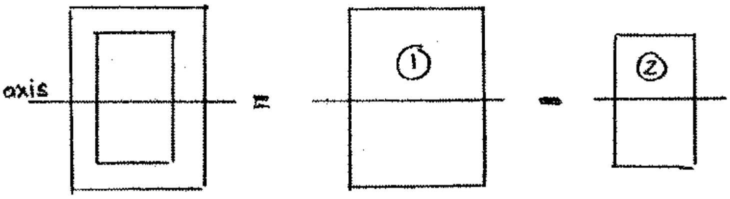
where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x \, dA = -\frac{My}{I} dA$$

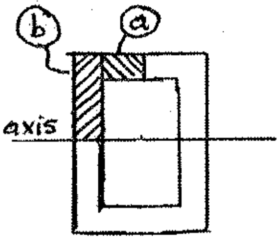
The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I}\int y \, dA = -\frac{M}{I}\bar{y}^* A^*$$

where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.



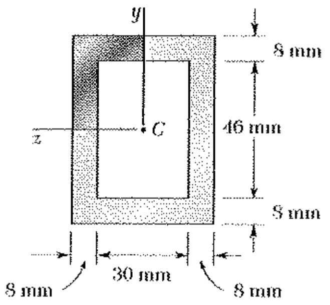
$$I = I_1 - I_2$$
$$= \tfrac{1}{12} b_1 h_1^3 - \tfrac{1}{12} b_2 h_2^3$$
$$= \tfrac{1}{12}(46)(62)^3$$
$$- \tfrac{1}{12}(30)(46)^3$$
$$= 0.67025 \times 10^6 \text{ mm}^4$$



$$\bar{y}^* A^* = \bar{y}_a A_a + \bar{y}_b A_b$$
$$= (27)(23)(8) + (11.5)(23)(8) = 7084 \text{ mm}^3$$

$$F = \frac{M\bar{y}^* A^*}{I} = \frac{(900)(7084 \times 10^{-9})}{0.67025 \times 10^{-6}} = 9.51 \text{ kN}$$ ◀

Problem 4.12



4.12 Solve Prob. 4.11, assuming that the beam is bent about a vertical axis and that the bending moment is 900 N · m.

4.11 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 900 N · m, determine the total force acting on the shaded portion of the beam.

The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$

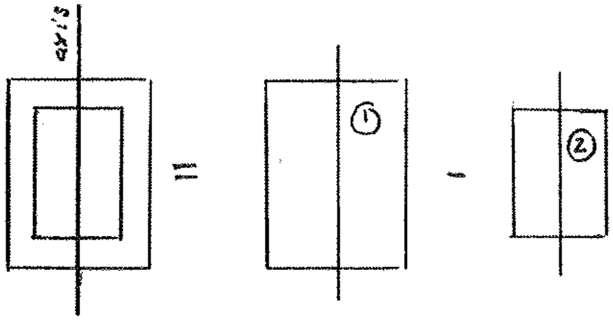
where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x \, dA = -\frac{My}{I} dA$$

The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I}\int y \, dA = -\frac{M}{I}\bar{y}^* A^*$$

where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.

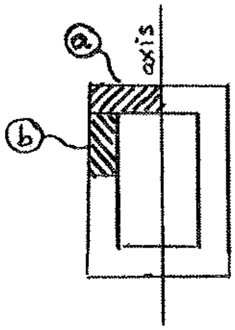


$$I = I_1 - I_2$$

$$= \frac{1}{12} b_1 h_1^3 - \frac{1}{12} b_2 h_2^3$$

$$= \frac{1}{12}(62)(46)^3 - \frac{1}{12}(46)(30)^3$$

$$= 0.399403 \times 10^6 \text{ mm}^4$$



$$\bar{y}^* A^* = \bar{y}_a A_a + \bar{y}_b A_b$$

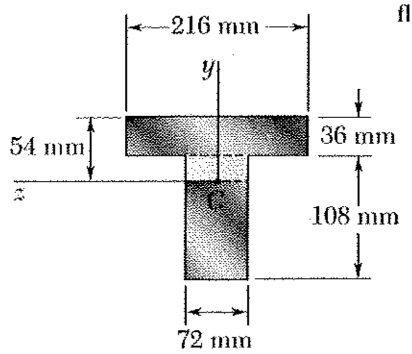
$$= (11.5)(8)(23) + (18)(23)(8)$$

$$= 5428 \text{ mm}^3.$$

$$F = \frac{M\bar{y}^* A^*}{I} = \frac{(900)(5428 \times 10^{-9})}{0.399403 \times 10^{-6}} = 12.2 \text{ kN.}$$

Problem 4.13

4.13 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 6 kN · m, determine the total force acting on the top flange.



The stress distribution over the entire cross-section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$

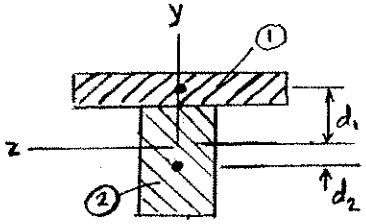
where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded portion is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x \, dA = -\frac{My}{I} dA$$

The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA_1 = -\frac{M}{I}\int y \, dA = -\frac{M}{I}\bar{y}^* A^*$$

where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.



$d_1 = 54 - 18 = 36$ mm

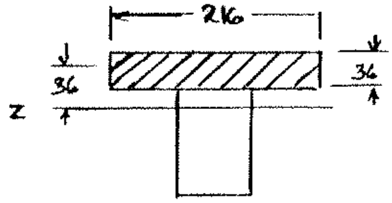
$d_2 = 54 - 36 - 54 = 36$mm

Moment of inertia of entire cross section.

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(216)(36)^3 + (216)(36)(36)^2 = 10.9175 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(72)(108)^3 + (72)(108)(36)^2 = 17.6360 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 = 28.5535 \times 10^6 \text{ mm}^4 = 28.5535 \times 10^{-6} \text{ m}^4$$



For the shade area

$A^* = (216)(36) = 7776$ mm^2

$\bar{y}^* = 36$ mm

$A^* \bar{y}^* = 279.936 \times 10^3 \text{ mm}^3 = 279.936 \times 10^{-6} \text{ m}^3$

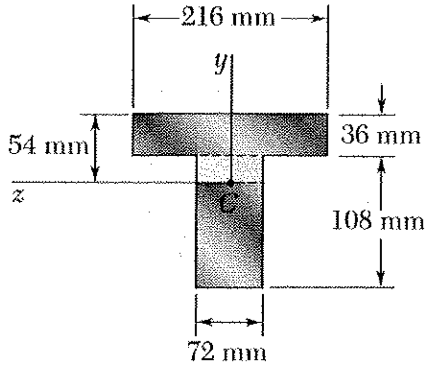
$$F = \left| \frac{M A^* \bar{y}^*}{I} \right| = \frac{(6 \times 10^3)(279.936 \times 10^{-6})}{28.5535 \times 10^{-6}}$$

$$= 58.8 \times 10^3 \text{ N}$$

F = 58.8 kN ◄

Problem 4.14

4.14 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 6 kN · m, determine the total force acting on the shaded portion of the web.



The stress distribution over the entire cross-section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$

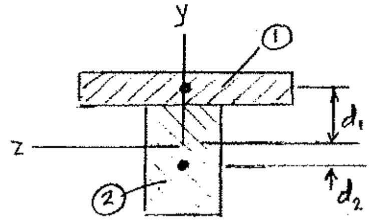
where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded portion is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x\, dA = -\frac{My}{I}\, dA$$

The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I}\, dA = -\frac{M}{I}\int y\, dA = -\frac{M}{I}\bar{y}^* A^*$$

where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.



$d_1 = 54 - 18 = 36$ mm

$d_2 = 54 + 36 - 54 = 36$mm

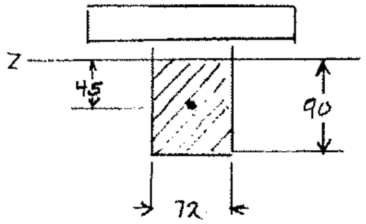
Moment of inertia of entire cross section.

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(216)(36)^3 + (216)(36)(36)^2 = 10.9175\times10^6\ \text{mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(72)(108)^3 + (72)(108)(36)^2 = 17.6360\times10^6\ \text{mm}^4$$

$$I = I_1 + I_2 = 28.5535\times10^6\ \text{mm}^4 = 28.5535\times10^{-6}\ \text{m}^4$$

For the shaded area



$$A^* = (72)(90) = 6480\ \text{mm}^2$$

$$\bar{y}^* = 45\ \text{mm}$$

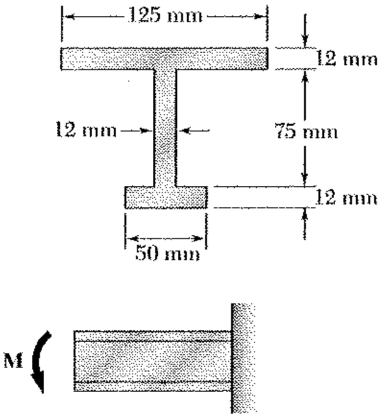
$$A^*\bar{y}^* = 291.6\times10^3\ \text{mm}^3 = 291.6\times10^{-6}\ \text{m}$$

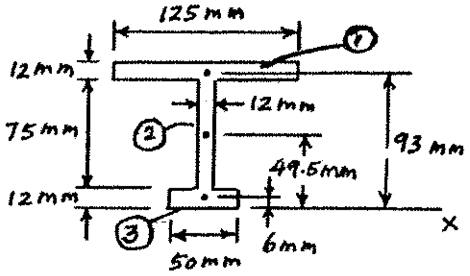
$$F = \left|\frac{MA^*\bar{y}^*}{I}\right| = \frac{(6\times10^3)(291.6\times10^{-6})}{28.5535\times10^{-6}}$$

$$= 61.3\times10^3\ \text{N} \qquad F = 61.3\ \text{kN}$$ ◄

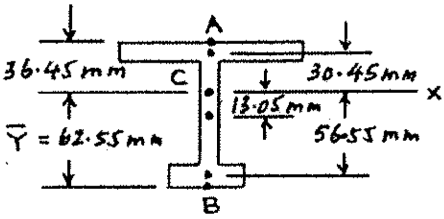
Problem 4.15

4.15 Knowing that for the casting shown the allowable stress is 42 MPa in tension and 105 MPa in compression, determine the largest couple **M** that can be applied.





	A, mm^2	$\bar{y}$, mm	$A\bar{y}, mm^3$
①	1500	93	139500
②	900	49.5	44550
③	600	6	3600
Σ	3000		187650



$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{187650}{3000} = 62.55 \text{ mm}$$

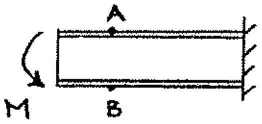
$$I_{x'} = \Sigma\left(\tfrac{1}{12}bh^3 + Ad^2\right)$$

$$= \tfrac{1}{12}(125)(12)^3 + (125)(12)(30.45)^2$$

$$+ \tfrac{1}{12}(12)(75)^3 + (12)(75)(13.05)^2$$

$$+ \tfrac{1}{12}(50)(12)^3 + (50)(12)(56.55)^2$$

$$= 3909900 \text{ mm}^4 = 3.91 \times 10^{-6} \text{ m}^4$$



ASSUME: $\sigma_A = +\sigma_{ALL} = +42$ MPa

$$M = \sigma_A \frac{I_{x'}}{c} = (42\times10^6)\frac{3.91\times10^{-6}}{0.03645} = 4505 \text{ Nm}$$

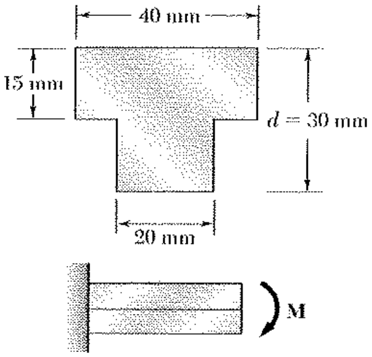
ASSUME: $\sigma_B = -\sigma_{ALL} = -6$

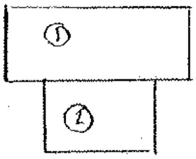
$$M = \sigma_B \frac{I_{x'}}{c} = (105\times10^6)\frac{3.91\times10}{0.06255} = 6563.5 \text{ Nm}$$

CHOOSE THE SMALLER M: $M_{ALL} = 4.5$ kNm ◀

Problem 4.16

4.16 The beam shown is made of a nylon for which the allowable stress is 24 MPa in tension and 30 MPa in compression. Determine the largest couple **M** that can be applied to the beam.





	A, mm^2	$\bar{y}_o$, mm	$A\bar{y}_o$, mm^3
①	600	22.5	13.5×10^3
②	300	7.5	2.25×10^3
Σ	900		15.75×10^3

$$\bar{Y}_o = \frac{15.5 \times 10^3}{900} = 17.5 \text{ mm}$$

The neutral axis lies 17.5mm above the bottom.

$y_{top} = 30 - 17.5 = 12.5 \text{ mm} = 0.0125 \text{ m}$, $y_{bot} = -17.5 \text{ mm} = -0.0175 \text{ m}$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(40)(15)^3 + (600)(5)^2 = 26.25 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(20)(15)^3 + (300)(10)^2 = 35.625 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 61.875 \times 10^3 \text{ mm}^4 = 61.875 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{My}{I}\right| \qquad M = \left|\frac{\sigma I}{y}\right|$$

Top: tension side. $M = \frac{(24 \times 10^6)(61.875 \times 10^{-9})}{0.0125} = 118.8 \text{ N}\cdot\text{m}$

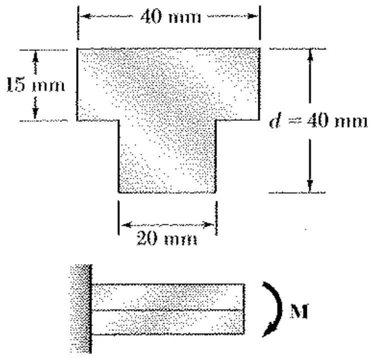
Bottom: compression. $M = \frac{(30 \times 10^6)(61.875 \times 10^{-9})}{0.0175} = 106.1 \text{ N}\cdot\text{m}$

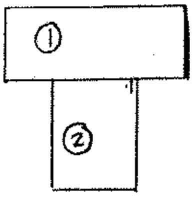
Choose smaller value. $M = 106.1 \text{ N}\cdot\text{m}$ ◀

Problem 4.17

4.17 Solve Prob. 4.16, assuming that $d = 40$ mm.

4.16 The beam shown is made of a nylon for which the allowable stress is 24 MPa in tension and 30 MPa in compression. Determine the largest couple **M** that can be applied to the beam.





	A, mm^2	$\bar{y}_0$, mm	$A\bar{y}_0$, mm^3
①	600	32.5	19.5×10^3
②	500	12.5	6.25×10^3
Σ	1100		25.75×10^3

$$\bar{Y}_0 = \frac{25.75 \times 10^3}{1100} = 23.41 \text{ mm}$$

The neutral axis lies 23.41 mm above the bottom.

$y_{top} = 40 - 23.41 = 16.59 \text{ mm} = 0.01659 \text{ m}$

$y_{bot} = -23.41 \text{ mm} = -0.02341 \text{ m}$

$$I_1 = \tfrac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \tfrac{1}{12}(40)(15)^3 + (600)(9.09)^2 = 60.827 \times 10^3 \text{ mm}^4$$

$$I_2 = \tfrac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \tfrac{1}{12}(20)(25)^3 + (500)(10.91)^2 = 85.556 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 146.383 \times 10^3 \text{ mm}^4 = 146.383 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{My}{I}\right| \qquad M = \left|\frac{\sigma I}{y}\right|$$

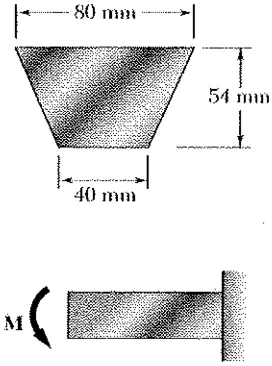
Top: tension side. $M = \dfrac{(24 \times 10^6)(146.383 \times 10^{-9})}{0.01659} = 212 \text{ N}\cdot\text{m}$

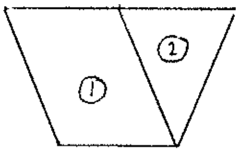
Bottom: compression. $M = \dfrac{(30 \times 10^6)(146.383 \times 10^{-9})}{0.02341} = 187.6 \text{ N}\cdot\text{m}$

Choose smaller value. $M = 187.6 \text{ N}\cdot\text{m}$ ◄

Problem 4.18

4.18 and *4.19* Knowing that for the extruded beam shown the allowable stress is 120 MPa in tension and 150 MPa in compression, determine the largest couple **M** that can be applied.





	A, mm^2	$\bar{y}_o$, mm	$A\bar{y}_o$, mm^3	d, mm
①	2160	27	58320	3
②	1080	36	38880	3
Σ	3240		97200	

$$\bar{Y} = \frac{97200}{3240} = 30 \text{ mm}$$

The neutral axis lies 30 mm above the bottom.

$$y_{top} = 54 - 30 = 24 \text{ mm} = 0.024 \text{ m}$$

$$y_{bot} = -30 \text{ mm} = -0.030 \text{ m}$$

$$I_1 = \tfrac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \tfrac{1}{12}(40)(54)^3 + (40)(54)(3)^2 = 544.32 \times 10^3 \text{ mm}^4$$

$$I_2 = \tfrac{1}{36} b_2 h_2^3 + A_2 d_2^2 = \tfrac{1}{36}(40)(54)^3 + \tfrac{1}{2}(40)(54)(6)^2 = 213.84 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 758.16 \times 10^3 \text{ mm}^4 = 758.16 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{My}{I}\right| \qquad |M| = \left|\frac{\sigma I}{y}\right|$$

top: tension side $\quad M = \dfrac{(120 \times 10^6)(758.16 \times 10^{-9})}{0.024} = 3.7908 \times 10^3$ N·m

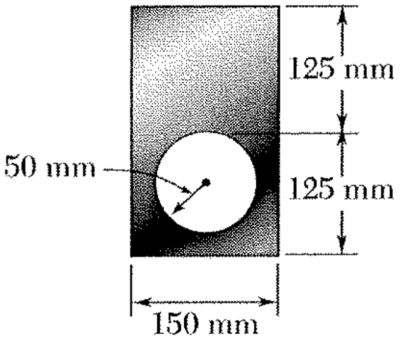
bottom: compression $\quad M = \dfrac{(150 \times 10^6)(758.16 \times 10^{-9})}{0.030} = 3.7908 \times 10^3$ N·m

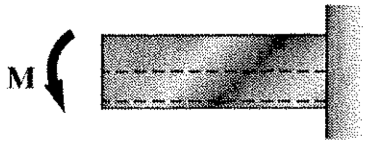
Choose the smaller as M_{all}. $\quad M_{all} = 3.7908 \times 10^3$ N·m

$M_{all} = 3.79$ kN·m ◀

Problem 4.19

4.18 and ***4.19*** Knowing that for the extruded beam shown the allowable stress is 120 MPa in tension and 150 MPa in compression, determine the largest couple **M** that can be applied.





① rectangle ② circular cutout

$A_1 = (150)(250) = 37.5\times10^3\ mm^2$

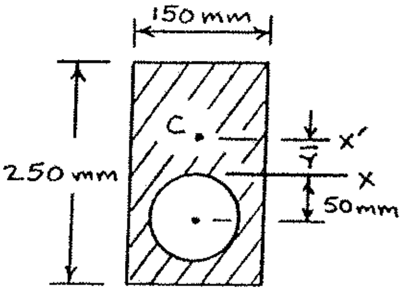
$A_2 = -\pi(50)^2 = 7.85398\times10^3\ mm^2$

$A = A_1 - A_2 = 29.64602\times10^3\ mm^2$

$\bar{y}_1 = 0\ mm$

$\bar{y}_2 = -50\ mm$

$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A}$



$$\bar{Y} = \frac{(37.5\times10^3)(0)+(-7.85393\times10^3(-50)}{29.64602\times10^3}$$

$= 13.2463\ mm$

$I_{x'} = \Sigma(I + Ad^2) = I_1 - I_2$

$= [\frac{1}{12}(150)(250)^3 + (37.5\times10^3)(13.2463)^2]$

$\quad - [\frac{\pi}{4}(50)^4 + (7.85398\times10^3)(50+13.2463)^2]$

$= 201.892\times10^6 - 36.3254\times10^6 = 165.567\times10^6\ mm^4 = 165.567\times10^{-6}\ m^4$

Top: (Tension side) $C = 125 - 13.2463 = 111.7537\ mm = 0.11175\ m$

$\sigma = \frac{Mc}{I} \qquad M = \frac{I\sigma}{c} = \frac{(165.567\times10^{-6})(120\times10^6)}{0.11175}$

$= 177.79\times10^3\ N\cdot m$

Bottom: (Compression side) $c = 125 + 13.2463 = 138.2463\ mm$

$= 0.13825\ m$

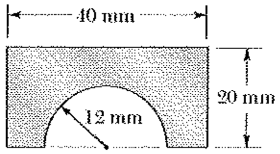
$\sigma = \frac{Mc}{I} \qquad M = \frac{I\sigma}{c} = \frac{(165.567\times10^{-6})(150\times10^6)}{0.13825}$

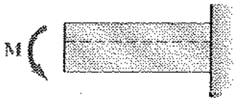
$= 179.64\times10^3\ N\cdot m$

Choose the smaller. $M = 177.8\times10^3\ N\cdot m$ $\quad M = 177.8\ kN\cdot m$ ◄

Problem 4.20

4.20 Knowing that for the beam shown the allowable stress is 84 MPa in tension and 110 MPa in compression, determine the largest couple **M** that can be applied.





① = rectangle ② = semi-circular cutout

$A_1 = (40)(20) = 800 \text{ mm}^2$

$A_2 = \frac{\pi}{2}(12)^2 = 226.2 \text{ mm}^2$

$A = 800 - 226.2 = 573.8 \text{ mm}^2$

$\bar{y}_1 = 10 \text{ mm}$ $\bar{y}_2 = \frac{4r}{3\pi} = \frac{(4)(12)}{3\pi} = 5.1 \text{ mm}$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{(800)(10) - (226.2)(5.1)}{573.8} = 11.9 \text{ mm}$$

Neutral axis lies 11.9 mm above the bottom

Moment of inertia about the base

$$I_b = \frac{1}{3}bh^3 - \frac{\pi}{8}r^4 = \frac{1}{3}(40)(20)^3 - \frac{\pi}{8}(12)^4 = 104075 \text{ mm}^4$$

Centroidal moment of inertia

$$\bar{I} = I_b - A\bar{Y}^2 = 104075 - (573.8)(11.9)^2$$
$$= 22819 \text{ mm}^4$$

$y_{top} = 20 - 11.9 = 8.1 \text{ mm}$, $y_{bot} = -11.9 \text{ mm}$

$|\sigma| = \left|\frac{My}{I}\right|$ $M = \left|\frac{\sigma I}{y}\right|$

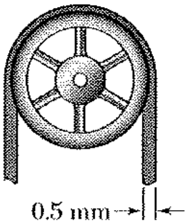
Top: tension side $M = \frac{(84 \times 10^6)(22819 \times 10^{-12})}{0.0081} = 236.6 \text{ Nm}$

Bottom: compression $M = \frac{(110 \times 10^6)(22819 \times 10^{-12})}{0.0119} = 210.9 \text{ Nm}$.

Choose the smaller value $M = 210.9 \text{ Nm}$ ◄

Problem 4.21

4.21 A steel band blade, that was originally straight, passes over 200 mm diameter pulleys when mounted on a band saw. Determine the maximum stress in the blade, knowing that it is 0.5 mm thick and 16 mm wide. Use $E = 200$ GPa.



Band blade thickness: $t = 0.5 mm$

Radius of pulley: $r = \frac{1}{2}d = 100 mm$

Radius of curvature of center line of blade:

$$\rho = r + \frac{1}{2}t = 100.25 mm$$

$$c = \frac{1}{2}t = 0.25 mm$$

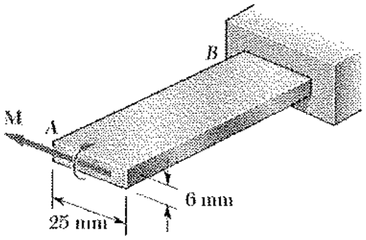
Maximum strain: $\varepsilon_m = \frac{c}{\rho} = \frac{0.25}{100.25} = 0.002494$

Maximum stress: $\sigma_m = E\varepsilon_m = (200 \times 10^9)(0.002494)$

$\sigma_m = 498.8 MPa$ $\qquad$ $\sigma_m = 498.8 MPa$ ◄

Problem 4.22

4.22 Knowing that $\sigma_{\text{all}} = 165$ MPa for the steel strip AB, determine (*a*) the largest couple **M** that can be applied, (*b*) the corresponding radius of curvature. Use $E = 200$ GPa.



$$I = \frac{1}{12} b h^3 = \left(\frac{1}{12}\right)(25)(6)^3 = 450 \text{ mm}^4$$

$$\sigma = \frac{Mc}{I} \qquad c = \left(\tfrac{1}{2}\right)(6) = 3 \text{ mm}$$

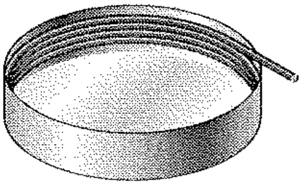
$$\text{(a)} \quad M = \frac{\sigma I}{c} = \frac{(165 \times 10^6)(450 \times 10^{-12})}{0.003}$$

$$M = 24.75 \text{ Nm} \quad ◄$$

$$\text{(b)} \quad \frac{c}{\rho} = \frac{\sigma_{max}}{E} \qquad \rho = \frac{Ec}{\sigma_{max}} = \frac{(220 \times 10^9)(0.003)}{165 \times 10^6} \qquad \rho = 4 \text{ m} \quad ◄$$

Problem 4.23

4.23 Straight rods of 6-mm diameter and 30-m length are stored by coiling the rods inside a drum of 1.25-m inside diameter. Assuming that the yield strength is not exceeded, determine (*a*) the maximum stress in a coiled rod, (*b*) the corresponding bending moment in the rod. Use $E = 200$ GPa.



Let D = inside diameter of the drum.
d = diameter of rod, $c = \frac{1}{2} d$,
ρ = radius of curvature of center line of rods when bent.

$$\rho = \tfrac{1}{2} D - \tfrac{1}{2} d = \tfrac{1}{2}(1.25) - \tfrac{1}{2}(6 \times 10^{-3}) = 0.622 \text{ m}$$

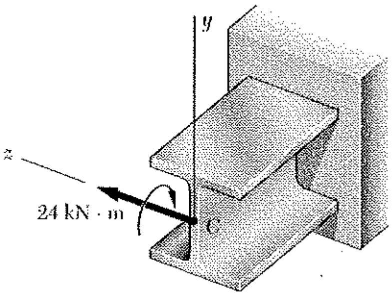
$$I = \frac{\pi}{4} c^4 = \frac{\pi}{4}(0.003)^4 = 63.617 \times 10^{-12} \text{ m}^4$$

$$\text{(a)} \quad \sigma_{max} = \frac{Ec}{\rho} = \frac{(200 \times 10^9)(0.003)}{0.622} = 965 \times 10^6 \text{ Pa} \qquad \sigma = 965 \text{ MPa} \quad ◄$$

$$\text{(b)} \quad M = \frac{EI}{\rho} = \frac{(200 \times 10^9)(63.617 \times 10^{-12})}{0.622} = 20.5 \text{ N-m} \qquad M = 20.5 \text{ N-m} \quad ◄$$

Problem 4.24

4.24 A 24 kN · m couple is applied to the W200 × 46.1 rolled-steel beam shown. (*a*) Assuming that the couple is applied about the *z* axis as shown, determine the maximum stress and the radius of curvature of the beam. (*b*) Solve part *a*, assuming that the couple is applied about the *y* axis. Use *E* = 200 GPa.



For W 200 × 46.1 rolled steel section

$I_x = 45.5 \times 10^6 \text{ mm}^4 = 45.5 \times 10^{-6} \text{ m}^4$

$S_x = 448 \times 10^3 \text{ mm}^3 = 448 \times 10^{-6} \text{ m}^3$

$I_y = 15.3 \times 10^6 \text{ mm}^4 = 15.3 \times 10^{-6} \text{ m}^4$

$S_y = 151 \times 10^3 \text{ mm}^3 = 151 \times 10^{-6} \text{ m}^3$

(a) $M_z = 24 \text{ kN·m} = 24 \times 10^3 \text{ N·m}$

$$\sigma = \frac{M}{S} = \frac{24 \times 10^3}{448 \times 10^{-6}} = 53.6 \times 10^6 \text{ Pa} = 53.6 \text{ MPa}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{24 \times 10^3}{(200 \times 10^9)(45.5 \times 10^{-6})} = 2.637 \times 10^{-3} \text{ m}^{-1}$$

$$\rho = 379 \text{ m}$$

(b) $M_y = 24 \text{ kN·m} = 24 \times 10^3 \text{ N·m}$

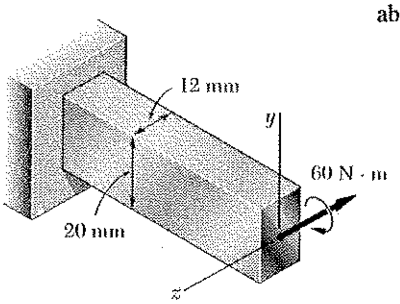
$$\sigma = \frac{M}{S} = \frac{24 \times 10^3}{151 \times 10^{-6}} = 158.9 \times 10^6 \text{ Pa} = 158.9 \text{ MPa}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{24 \times 10^3}{(200 \times 10^9)(15.3 \times 10^{-6})} = 7.84 \times 10^{-3} \text{ m}^{-1}$$

$$\rho = 127.5 \text{ m}$$

Problem 4.25

4.25 A 60 N · m couple is applied to the steel bar shown. (*a*) Assuming that the couple is applied about the *z* axis as shown, determine the maximum stress and the radius of curvature of the bar. (*b*) Solve part *a*, assuming that the couple is applied about the *y* axis. Use $E = 200$ GPa.



(a) Bending about z-axis.

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(12)(20)^3 = 8 \times 10^3 \text{ mm}^4$$
$$= 8 \times 10^{-9} \text{ m}^4$$

$$c = \frac{20}{2} = 10 \text{ mm} = 0.010 \text{ m}$$

$$\sigma = \frac{Mc}{I} = \frac{(60)(0.010)}{8 \times 10^{-9}} = 75.0 \times 10^6 \text{ Pa} \qquad \sigma = 75.0 \text{ MPa} \blacktriangleleft$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{60}{(200 \times 10^9)(8 \times 10^{-9})} = 37.5 \times 10^{-3} \text{ m}^{-1} \qquad \rho = 26.7 \text{ m} \blacktriangleleft$$

(b) Bending about y-axis.

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(20)(12)^3 = 2.88 \times 10^3 \text{ mm}^4 = 2.88 \times 10^{-9} \text{ m}^4$$

$$c = \frac{12}{2} = 6 \text{ mm} = 0.006 \text{ m}$$

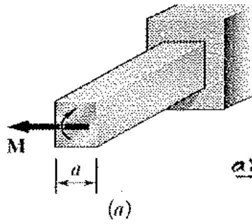
$$\sigma = \frac{Mc}{I} = \frac{(60)(0.006)}{2.88 \times 10^{-9}} = 125.0 \times 10^6 \text{ Pa} \qquad \sigma = 125.0 \text{ MPa} \blacktriangleleft$$

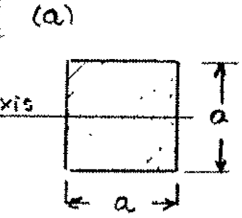
$$\frac{1}{\rho} = \frac{M}{EI} = \frac{60}{(200 \times 10^9)(2.88 \times 10^{-9})} = 104.17 \times 10^{-3} \text{ m}^{-1}$$

$$\rho = 9.60 \text{ m} \blacktriangleleft$$

Problem 4.26

4.26 A couple of magnitude *M* is applied to a square bar of side *a*. For each of the orientations shown, determine the maximum stress and the curvature of the bar.



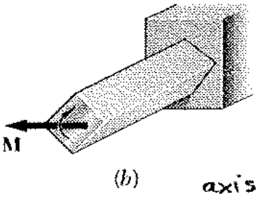


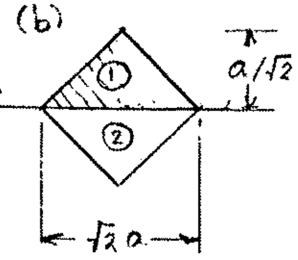
$$I = \frac{1}{12} bh^3 = \frac{1}{12} a a^3 = \frac{a^4}{12}$$

$$c = \frac{a}{2}$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{M \frac{a}{2}}{\frac{a^4}{12}} \qquad \sigma_{max} = \frac{6M}{a^3}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{M}{E\frac{a^4}{12}} \qquad \frac{1}{\rho} = \frac{12M}{Ea^4}$$





For one triangle, the moment of inertia about its base is

$$I_1 = \frac{1}{12} bh^3 = \frac{1}{12}(\sqrt{2}a)\left(\frac{a}{\sqrt{2}}\right)^3 = \frac{a^4}{24}$$

$$I_2 = I_1 = \frac{a^4}{24}$$

$$I = I_1 + I_2 = \frac{a^4}{12}$$

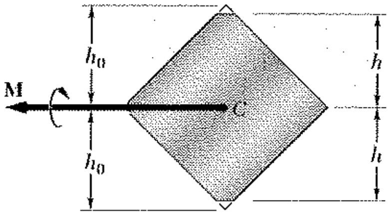
$$c = \frac{a}{\sqrt{2}}$$

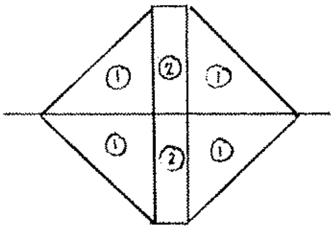
$$\sigma_{max} = \frac{Mc}{I} = \frac{M a/\sqrt{2}}{a^4/12} = \frac{6\sqrt{2}M}{a^3} \qquad \sigma_{max} = \frac{8.49M}{a^3}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{M}{E\frac{a^4}{12}} \qquad \frac{1}{\rho} = \frac{12M}{Ea^4}$$

Problem 4.27

4.27 A portion of a square bar is removed by milling, so that its cross section is as shown. The bar is then bent about its horizontal axis by a couple **M**. Considering the case where $h = 0.9h_0$, express the maximum stress in the bar in the form $\sigma_m = k\sigma_0$ where σ_0 is the maximum stress that would have occurred if the original square bar had been bent by the same couple **M**, and determine the value of k.





$$I = 4I_1 + 2I_2$$

$$= (4)(\tfrac{1}{12})h\,h^3 + (2)(\tfrac{1}{3})(2h_0 - 2h)(h^3)$$

$$= \tfrac{1}{3}h^4 + \tfrac{4}{3}h_0h^3 - \tfrac{4}{3}h\,h^3 = \tfrac{4}{3}h_0h^3 - h^4$$

$$c = h$$

$$\sigma_1 = \frac{Mc}{I} = \frac{Mh}{\frac{4}{3}h_0h^3 - h^4} = \frac{3M}{(4h_0 - 3h)h^2}$$

For the original square, $h = h_0$, $c = h_0$

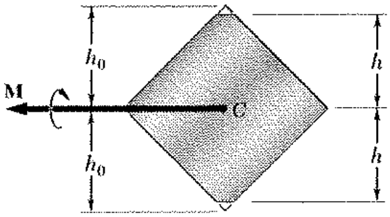
$$\sigma_0 = \frac{3M}{(4h_0 - 3h_0)h_0^2} = \frac{3M}{h_0^3}$$

$$\frac{\sigma}{\sigma_0} = \frac{h_0^3}{(4h_0 - 3h)h^2} = \frac{h_0^3}{(4h_0 - (3)(0.9)h_0)(0.9h_0^2)} = 0.950$$

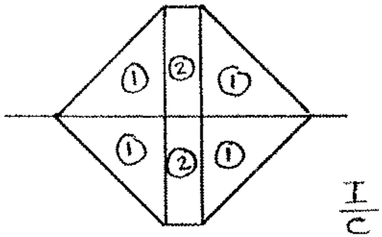
$\sigma = 0.950\,\sigma_0$ $\qquad$ $k = 0.950$ ◄

Problem 4.28

4.28 In Prob. 4.27, determine (*a*) the value of h for which the maximum stress σ_m is as small as possible, (*b*) the corresponding value of k.



4.27 A portion of a square bar is removed by milling, so that its cross section is as shown. The bar is then bent about its horizontal axis by a couple **M**. Considering the case where $h = 0.9h_0$, express the maximum stress in the bar in the form $\sigma_m = k\sigma_0$ where σ_0 is the maximum stress that would have occurred if the original square bar had been bent by the same couple **M**, and determine the value of k.



$$I = 4I_1 + 2I_2$$

$$= (4)(\tfrac{1}{12})h h^3 + (2)(\tfrac{1}{3})(2h_0 - 2h)h^3$$

$$= \tfrac{1}{3}h^4 - \tfrac{4}{3}h_0 h^3 - \tfrac{4}{3}h^3 = \tfrac{4}{3}h_0 h^3 - h^4$$

$$c = h \qquad \frac{I}{c} = \tfrac{4}{3}h_0 h^2 - h^3$$

$\frac{I}{c}$ is maximum at $\frac{d}{dh}\left[\tfrac{4}{3}h_0 h^2 - h^3\right] = 0$

$$\tfrac{8}{3}h_0 h - 3h^2 = 0 \qquad h = \tfrac{8}{9}h_0$$

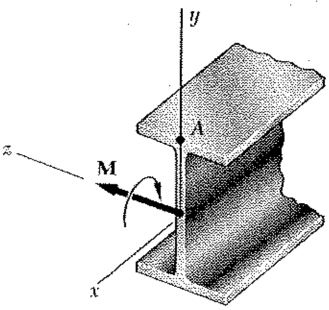
$$\frac{I}{c} = \tfrac{4}{3}h_0\left(\tfrac{8}{9}h_0\right)^2 - \left(\tfrac{8}{9}h_0\right)^3 = \frac{256}{729}h_0^3 \qquad \sigma = \frac{Mc}{I} = \frac{729}{256}\frac{M}{h_0^3}$$

For the original square, $h = h_0 \quad c = h_0 \quad \frac{I_0}{c_0} = \frac{1}{3}h_0^3$

$$\sigma_0 = \frac{Mc_0}{I_0} = \frac{3M}{h_0^2}$$

$$\frac{\sigma}{\sigma_0} = \frac{729}{256}\cdot\frac{1}{3} = \frac{729}{768} \approx 0.949 \qquad k = 0.949$$

Problem 4.29



4.29 A W200 × 31.3 rolled-steel beam is subjected to a couple **M** of moment 45 kN · m. Knowing that $E = 200$ GPa and $\nu = 0.29$, determine (*a*) the radius of curvature ρ, (*b*) the radius of curvature ρ' of a transverse cross section.

For W 200 × 31.3 rolled steel section

$$I = 31.4 \times 10^6 \text{ mm}^4 = 31.4 \times 10^{-6} \text{ m}^4$$

(a) $$\frac{1}{\rho} = \frac{M}{EI} = \frac{45 \times 10^8}{(200 \times 10^9)(31.4 \times 10^{-6})} = 7.17 \times 10^{-3} \text{ m}^{-1}$$

$\rho = 139.6$ m

(b) $$\frac{1}{\rho'} = \nu \frac{1}{\rho} = (0.29)(7.17 \times 10^{-3}) = 2.07 \times 10^{-3} \text{ m}^{-1} \qquad \rho' = 481 \text{ m}$$

Problem 4.30

4.30 For the bar and loading of Example 4.01, determine (*a*) the radius of curvature ρ, (*b*) the radius of curvature ρ' of a transverse cross section, (*c*) the angle between the sides of the bar that were originally vertical. Use $E = 200$ GPa and $\nu = 0.29$.

From Example 4.01 $M = 3$ kNm, $I = 360000 \text{ mm}^4$

(a) $$\frac{1}{\rho} = \frac{M}{EI} = \frac{(3 \times 10^3)}{(200 \times 10^9)(36 \times 10^{-8})} = 0.04167 \text{ m}^{-1} \qquad \rho = 24 \text{ m}$$

(b) $$\varepsilon' = \nu \varepsilon = \frac{\nu c}{\rho} = \nu \frac{c}{\rho'}$$

$$\frac{1}{\rho'} = \nu \frac{1}{\rho} = (0.29)(0.04167) \text{ in}^{-1} = 0.01208 \text{ m}^{-1} \qquad \rho' = 82.75 \text{ m.}$$

(c) $$\theta = \frac{\text{length of arc}}{\text{radius}} = \frac{b}{\rho'} = \frac{0.02}{82.75} = 241.7 \times 10^{-6} \text{ rad} = 0.01380°$$

Problem 4.31

4.31 For the aluminum bar and loading of Sample Problem 4.1, determine (*a*) the radius of curvature ρ' of a transverse cross section, (*b*) the angle between the sides of the bar that were originally vertical. Use $E = 73$ GPa and $\nu = 0.33$.

From Sample Problem 4.1 $I = 4.38 \times 10^6 \text{ mm}^4$ $M = 10.1$ kN

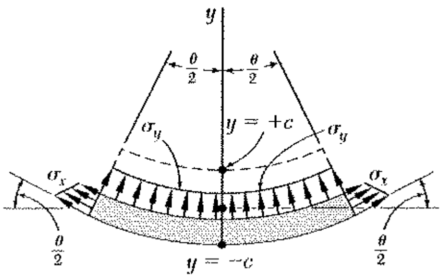
$$\frac{1}{\rho} = \frac{M}{EI} = \frac{10.1 \times 10^3}{(73 \times 10^9)(4.38 \times 10^{-6})} = 0.03159 \text{ m}^{-1}$$

(a) $$\frac{1}{\rho'} = \nu \frac{1}{\rho} = (0.33)(0.03159) = 0.010424 \text{ m}^{-1}$$

$\rho' = 95.9$ mm

(b) $$\theta = \frac{\text{length of arc}}{\text{radius}} = \frac{b}{\rho'} = \frac{0.08}{95.9} = 834.2 \times 10^{-6} \text{ rad} = 0.0478°$$

Problem 4.32



4.32 It was assumed in Sec. 4.3 that the normal stresses σ_y in a member in pure bending are negligible. For an initially straight elastic member of rectangular cross section, (*a*) derive an approximate expression for σ_y as a function of y, (*b*) show that $(\sigma_y)_{\max} = -(c/2\rho)(\sigma_x)_{\max}$ and, thus, that σ_y can be neglected in all practical situations. (*Hint*: Consider the free-body diagram of the portion of beam located below the surface of ordinate y and assume that the distribution of the stress σ_x is still linear.)

Denote the width of the beam by b and the length by L.

$$\theta = \frac{L}{\rho}$$

Using the free body diagram above, with $\cos\frac{\theta}{2} \approx 1$

$$\Sigma F_y = 0 \qquad \sigma_y b L + 2\int_{-c}^{y} \sigma_x \, b\, dy \, \sin\frac{\theta}{2} = 0$$

$$\sigma_y = -\frac{2}{L}\sin\frac{\theta}{2}\int_{-c}^{y}\sigma_x\,dy \approx -\frac{\theta}{L}\int_{-c}^{y}\sigma_x\,dy = -\frac{1}{\rho}\int_{-c}^{y}\sigma_x\,dy$$

But $\quad \sigma_x = -(\sigma_x)_{\max}\dfrac{y}{c}$

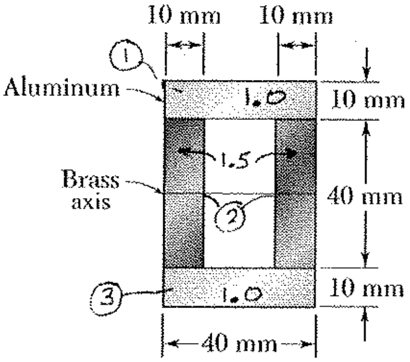
(a) $$\sigma_y = \frac{(\sigma_x)_{\max}}{\rho c}\int_{-c}^{y} y\,dy = \frac{(\sigma_x)_{\max}}{\rho c}\left.\frac{y^2}{2}\right|_{-c}^{y} = \frac{(\sigma_x)_{\max}}{2\rho c}(y^2 - c^2)$$

The maximum value σ_y occurs at $y = 0$

(b) $$(\sigma_y)_{\max} = -\frac{(\sigma_x)_{\max}c^2}{2\rho c} = -\frac{(\sigma_x)_{\max}c}{2\rho}$$

Problem 4.33

4.33 and ***4.34*** A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.



	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

Use aluminum as the reference material.

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the figure.

For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1.0}{12}(40)(10)^3 + (1.0)(40)(10)(25)^2 = 253.333 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{1.5}{12}(20)(40)^3 = 160 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 253.333 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 666.67 \times 10^3 \text{ mm}^4 = 666.67 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \left|\frac{\sigma I}{ny}\right|$$

Aluminum: $n = 1.0$, $|y| = 30 \text{ mm} = 0.030 \text{ m}$, $\sigma = 100 \times 10^6$ Pa

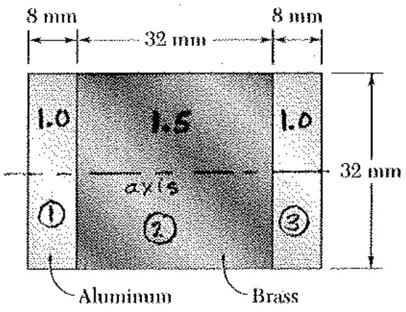
$$M = \frac{(100 \times 10^6)(666.67 \times 10^{-9})}{(1.0)(0.030)} = 2.2222 \times 10^3 \text{ N·m}$$

Brass: $n = 1.5$ $|y| = 20 \text{ mm} = 0.020 \text{ m}$, $\sigma = 160 \times 10^6$ Pa

$$M = \frac{(160 \times 10^6)(666.67 \times 10^{-9})}{(1.5)(0.020)} = 3.5556 \times 10^3 \text{ N·m}$$

Choose the smaller value. $M = 2.22 \times 10^3$ N·m $\qquad$ $M = 2.22$ kN·m ◄

Problem 4.34



4.33 and *4.34* A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.

	Aluminum	**Brass**
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

Use aluminum as the reference material.

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the figure.

For the transformed section

$$I_1 = \frac{n_1}{12} b_1 h_1^3 = \frac{1.0}{12}(8)(32)^3 = 21.8453 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{1.5}{12}(32)(32)^3 = 131.072 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 21.8453 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 174.7626 \times 10^3 \text{ mm}^4 = 174.7626 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \left|\frac{\sigma I}{ny}\right|$$

Aluminum: $n = 1.0$, $|y| = 16 \text{ mm} = 0.016 \text{ m}$, $\sigma = 100 \times 10^6$ Pa

$$M = \frac{(100 \times 10^6)(174.7626 \times 10^{-9})}{(1.0)(0.016)} = 1.0923 \times 10^3 \text{ N}\cdot\text{m}$$

Brass: $n = 1.5$, $|y| = 16 \text{ mm} = 0.016 \text{ m}$, $\sigma = 160 \times 10^6$ Pa

$$M = \frac{(160 \times 10^6)(174.7626 \times 10^{-9})}{(1.5)(0.016)} = 1.1651 \times 10^3 \text{ N}\cdot\text{m}$$

Choose the smaller value. $M = 1.092 \times 10^3$ N·m $\quad$ $M = 1.092$ kN·m ◀

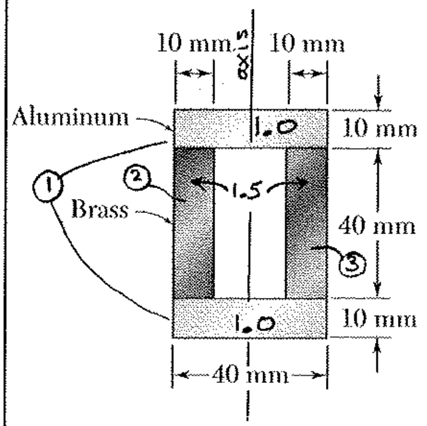
Problem 4.35

4.35 and *4.36* For the composite bar indicated, determine the largest permissible bending moment when the bar is bent about a vertical axis.

4.35 Bar of Prob. 4.33.

4.33 and *4.34* A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.

	Aluminum	**Brass**
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa



Use aluminum as the reference material.

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the figure.

For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 = \frac{1.0}{12}(20)(40^3) = 106.6667 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.5}{12}(40)(10)^3 + (1.5)(40)(10)(15)^2 = 140 \times 10^3 \text{ mm}^4$$

$$I_3 = I_2 = 140 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 386.667 \times 10^3 \text{ mm}^4 = 385.667 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \left|\frac{\sigma I}{ny}\right|$$

Aluminum: $n = 1.0$, $|y| = 20 \text{ mm} = 0.020$, $\sigma = 100 \times 10^6$ Pa

$$M = \frac{(100 \times 10^6)(386.667 \times 10^{-9})}{(1.0)(0.020)} = 1933 \text{ N}\cdot\text{m}$$

Brass: $n = 1.5$, $|y| = 20 \text{ mm} = 0.020 \text{ m}$, $\sigma = 160 \times 10^6$ Pa

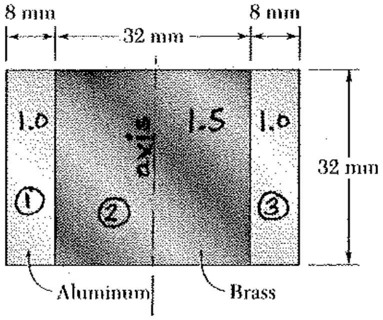
$$M = \frac{(160 \times 10^6)(386.667 \times 10^{-9})}{(1.5)(0.020)} = 2062 \text{ N}\cdot\text{m}$$

Choose the smaller value. $M = 1933$ N·m $M = 1.933$ kN·m ◀

Problem 4.36

4.35 and *4.36* For the composite bar indicated, determine the largest permissible bending moment when the bar is bent about a vertical axis.

4.36 Bar of Prob. 4.34.



	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

Use aluminum as the reference material

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the figure.

For the transformed section

$$I_1 = \frac{n_1}{12} h_1 b_1^3 + n_1 A_1 d_1^2 = \frac{1.0}{12}(32)(8)^3 + (1.0)[(32)(8)](20)^2 = 103.7653 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} h_2 b_2^3 = \frac{1.5}{12}(32)(32)^3 = 131.072 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 103.7653 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 338.58 \times 10^3 \text{ mm}^4 = 338.58 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \frac{nMy}{I} \qquad M = \left|\frac{\sigma I}{ny}\right|$$

Aluminum: $n = 1.0$, $|y| = 24 \text{ mm} = 0.024 \text{ m}$, $\sigma = 100 \times 10^6$ Pa

$$M = \frac{(100 \times 10^6)(338.58 \times 10^{-9})}{(1.0)(0.024)} = 1.411 \times 10^3 \text{ N}\cdot\text{m}$$

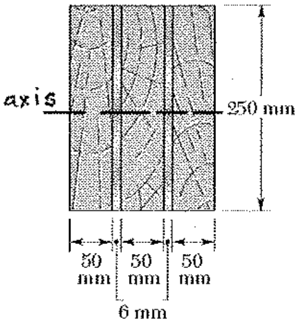
Brass: $n = 1.5$, $|y| = 16 \text{ mm} = 0.016 \text{ m}$, $\sigma = 160 \times 10^6$ Pa

$$M = \frac{(160 \times 10^6)(338.58 \times 10^{-9})}{(1.5)(0.016)} = 2.257 \times 10^3 \text{ N}\cdot\text{m}$$

Choose the smaller value. $M = 1.411 \times 10^3$ N·m $\qquad$ $M = 1.411$ kN·m ◄

Problem 4.37

4.37 Three wooden beams and two steel plates are securely bolted together to form the composite member shown. Using the data given below, determine the largest permissible bending moment when the member is bent about a horizontal axis.



	Wood	Steel
Modulus of elasticity	14 GPa	200 GPa
Allowable stress	14 MPa	150 MPa

Use wood as the reference material

For wood $n_w = 1$

For steel $n = E_s/E_w = 200/14 = 14.29$

Properties of the geometric section.

Steel: $I_s = \frac{1}{12}(6+6)(250)^3 = 15.625 \times 10^6 \text{ mm}^4$.

Wood: $I_w = \frac{1}{12}(50+50+50)(250^3) = 195.3125 \times 10^6 \text{ mm}^4$.

Transformed section

$$I_{trans} = n_s I_s + n_w I_w = (14.29)(15.625 \times 10^6) + (1)(195.3125 \times 10^6)$$
$$= 418.6 \times 10^6 \text{ mm}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \left|\frac{\sigma I}{ny}\right|$$

Wood: $n = 1$ $\quad |y| = 125 \text{ mm}$ $\quad \sigma = 14 \text{ MPa}$.

$$M = \frac{(14 \times 10^6)(418.6 \times 10^{-6})}{(1)(0.125)} = 46.9 \times 10^3 \text{ Nm}$$

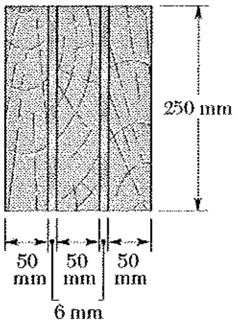
Steel: $n = 14.29$ $\quad |y| = 125 \text{ mm}$ $\quad \sigma = 150 \text{ MPa}$

$$M = \frac{(150 \times 10^6)(418.6 \times 10^{-6})}{(14.29)(0.125)} = 35.2 \times 10^3 \text{ Nm}$$

Choose the smaller value $\quad M = 35.2 \times 10^3 \text{ Nm}$

$M = 35.2 \text{ kN.m}$ ◄

Problem 4.38



4.38 For the composite member of Prob. 4.37, determine the largest permissible bending moment when the member is bent about a vertical axis.

4.37 Three wooden beams and two steel plates are securely bolted together to form the composite member shown. Using the data given below, determine the largest permissible bending moment when the member is bent about a horizontal axis.

	Wood	Steel
Modulus of elasticity	14 GPa	200 GPa
Allowable stress	14 MPa	150 MPa

Use wood as the reference material

For wood $n_w = 1$

For steel $n_s = E_s/E_w = 200/14 = 14.29$.

Properties of the geometric section

Total: $I_t = \frac{1}{12} hb^3 = \frac{1}{12}(250)(162)^3 = 88573500 \text{ mm}^4$

Steel: $I_s = \frac{1}{12} h(B^3 - b^3) = \frac{1}{12}(250)(62^3 - 50^3) = 2361000 \text{ mm}^4$

Wood: $I_w = I_t - I_s = 86212500 \text{ mm}^4$

Transformed section

$$I_{trans} = n_s I_s + n_w I_w = (14.29)(2361000) + (1)(86212500) = 120 \times 10^6 \text{ mm}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \left|\frac{\sigma I}{ny}\right|$$

Wood: $n = 1$ $\quad |y| = 81 \text{ mm}$ $\quad \sigma = 14 \text{ MPa}$

$$M = \frac{(14\times10^6)(120\times10^{-6})}{(1)(0.081)} = 20.74 \times 10^3 \text{ N.m}$$

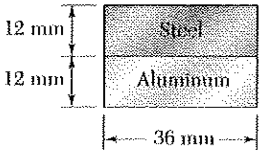
Steel: $n = 14.29$ $\quad |y| = 31 \text{ mm}$ $\quad \sigma = 150 \text{ MPa}$

$$M = \frac{(150\times10^6)(120\times10^{-6})}{(14.29)(0.031)} = 40.6 \times 10^3 \text{ N.m}$$

Choose the smaller value. $M = 20.74 \text{ kNm}$

$M = 20.74 \text{ kNm}$ ◀

Problem 4.39

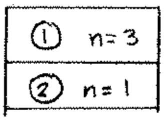


4.39 and 4.40 A steel bar (E_s = 210 GPa) and an aluminum bar (E_a = 70 GPa) are bonded together to form the composite bar shown. Determine the maximum stress in (*a*) the aluminum, (b) the steel, when the bar is bent about a horizontal axis, with M = 200 N · m.

Use aluminum as the reference material.

For aluminum $n = 1$

For steel $n = E_s/E_a = 210/70 = 3$



Transformed section

	A, mm^2	nA, mm^2	$\bar{y}_o, mm$	$nA\bar{y}_o, mm^3$
①	432	1296	18	23328
②	432	432	6	2592
Σ		1728		25920

$$\bar{Y}_o = \frac{25920}{1728} = 15 \text{ mm}$$

The neutral axis lies 15mm above the bottom.

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{3}{12}(36)(12)^3 + (1296)(3)^2 = 27.216 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1}{12}(36)(12)^3 + (432)(9)^2 = 40.176 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 67.392 \times 10^3 \text{ mm}^4 = 67.392 \times 10^{-9} \text{ m}^4$$

$$M = 200 \text{ N}\cdot\text{m}$$

$$\sigma = -\frac{nMy}{I}$$

(a) Aluminum: $n = 1$, $y = -15 \text{ mm} = -0.015 \text{ m}$

$$\sigma_a = -\frac{(1)(200)(-0.015)}{67.392 \times 10^{-9}} = 44.516 \times 10^6 \text{ Pa}$$

$\sigma_a = 44.5$ MPa ◄

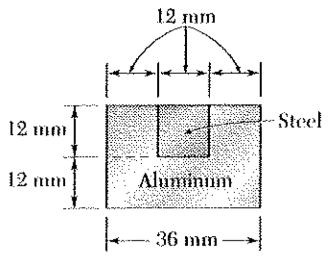
(b) Steel: $n = 3$, $y = 9 \text{ mm} = 0.009 \text{ m}$

$$\sigma_s = -\frac{(3)(200)(0.009)}{67.392 \times 10^{-9}} = -80.128 \times 10^6 \text{ Pa}$$

$\sigma_s = -80.1$ MPa ◄

Problem 4.40

4.39 and 4.40 A steel bar (E_s = 210 GPa) and an aluminum bar (E_a = 70 GPa) are bonded together to form the composite bar shown. Determine the maximum stress in (*a*) the aluminum, (b) the steel, when the bar is bent about a horizontal axis, with M = 200 N · m.

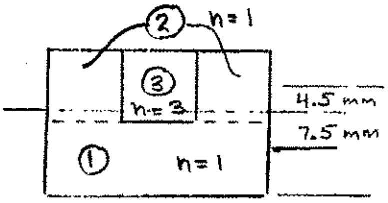


Use aluminum as the reference material.

For aluminum, $n = 1$

For steel, $n = E_s/E_a = 210/70 = 3$

Transformed section.



	A, mm²	nA, mm²	$\bar{y}_o$, mm	$nA\bar{y}_o$, mm³
①	432	432	6	2592
②	288	288	18	5184
③	144	432	18	7776
		1152		15552

$\bar{Y}_o = \dfrac{15552}{1152} = 13.5$ mm The neutral axis lies 13.5 mm above the bottom.

$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1}{12}(36)(12)^3 + (432)(7.5)^2 = 29.484 \times 10^3 \text{ mm}^4$

$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1}{12}(24)(12)^3 + (288)(4.5)^2 = 9.288 \times 10^3 \text{ mm}^4$

$I_3 = \frac{n_3}{12} b_3 h_3^3 + n_3 A_3 d_3^2 = \frac{3}{12}(12)(12)^3 + (432)(4.5)^2 = 13.932 \times 10^3 \text{ mm}^4$

$I = I_1 + I_2 + I_3 = 52.704 \times 10^3 \text{ mm}^4 = 52.704 \times 10^{-9} \text{ m}^4$

$M = 60$ N·m

$\sigma = -\dfrac{nMy}{I}$

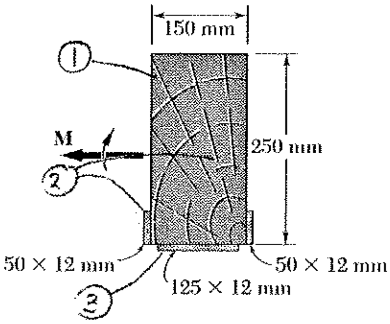
(a) Aluminum: $n = 1$, $y = -13.5$ mm $= -0.0135$ m

$\sigma_a = -\dfrac{(1)(200)(-0.0135)}{52.704 \times 10^{-9}} = 51.2 \times 10^6$ Pa $\sigma_a = 51.2$ MPa ◀

(b) Steel: $n = 3$, $y = 10.5$ mm $= 0.0105$ m

$\sigma_s = -\dfrac{(3)(200)(0.0105)}{52.704 \times 10^{-9}} = -119.5 \times 10^6$ Pa $\sigma_s = -119.5$ MPa ◀

Problem 4.41



4.41 and *4.42* The 150 × 250-mm timber beam has been strengthened by bolting to it the steel reinforcement shown. The modulus of elasticity for wood is 12 GPa and for steel 200 GPa. Knowing that the beam is bent about a horizontal axis by a couple of moment $M = 50$ kN · m, determine the maximum stress in (*a*) the wood, (*b*) the steel.

Use wood as the reference material.

For wood, $n = 1$

For steel, $n = \frac{E_s}{E_w} = \frac{200}{12} = 16.667$

	A, mm²	nA, mm²	$\bar{y}$, mm	$nA\bar{y}$, mm³
①	37500	37500	137	5137500
②	1200	20000·4	37	740014·8
③	1500	25000·5	6	150003
		82500·9		6027517·8

Transformed section.

$$\bar{Y} = \frac{6027517.8}{82500.9}$$

$$= 73 \text{ mm}$$

The neutral axis lies 73 mm above the bottom.

Distances from neutral axis.

$d_1 = 137 - 73 = 64 \text{mm}$

$d_2 = |37 - 73| = 36 \text{ mm}$

$d_3 = |6 - 73| = 67 \text{ mm}$

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1}{12}(150)(250)^3 + (37500)(64)^2 = 348.91 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{16.667}{12}(24)(50)^3 + (20000.4)(36)^2 = 30.09 \times 10^6 \text{ mm}^4$$

$$I_3 = \frac{n_3}{12} b_3 h_3^3 + n_3 A_3 d_3^2 = \frac{16.667}{12}(125)(12)^3 + (25000.5)(67)^2 = 112.53 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 491.53 \times 10^6 \text{ mm}^4 = 491.53 \times 10^{-6} \text{ m}^4$$

$M = 50$ kNm

(a) Wood: $n = 1$, $y = 262 - 73 = 189 \text{ mm} = 0.189 \text{ m}$

$$\sigma_w = -\frac{nMy}{I} = -\frac{(1)(50\times10^3)(0.189)}{491.53\times10^{-6}} = -19.225 \text{ MPa}$$

$\sigma_w = -19.2$ MPa ◄

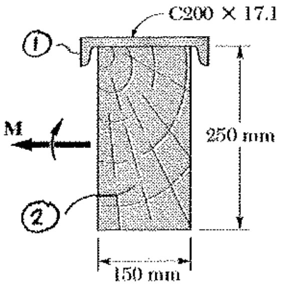
(b) Steel: $n = 16.667$, $y = -0.073$ m

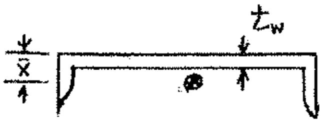
$$\sigma_s = -\frac{nMy}{I} = -\frac{(16.667)(50\times10^3)(-0.073)}{491.53\times10^{-6}} = 123.77 \text{ MPa}$$

$\sigma_s = 123.8$ MPa ◄

Problem 4.42

4.41 and 4.42 The 150 × 250 mm timber beam has been strengthened by bolting to it the steel reinforcement shown. The modulus of elasticity for wood is 12 GPa and for steel 200 GPa. Knowing that the beam is bent about a horizontal axis by a couple of moment $M = 50$ kN · m, determine the maximum stress in (*a*) the wood, (*b*) the steel.





Use wood as the reference material.

For wood $n = 1$

For steel $n = E_s / E_w = 200/12 = 16.67$

For C8×11.5 channel section, $A = 2170\ \text{mm}^2$

$t_w = 5.6\ \text{mm}$, $\bar{x} \doteq 14.4\ \text{mm}$, $I_y = 538000\ \text{mm}$

For the composite section the centroid of the channel lies $250 + 5.6 - 14.4 = 241.2$ mm above the base. $\bar{y}_o = 241.2$ mm for channel.

Transformed section

$$\bar{Y}_o = \frac{13412669}{73674} = 182.1\ \text{mm}$$

The neutral axis lies 182.1 mm above the bottom.

	A, mm²	nA, mm²	$\bar{y}_o$, mm	$nA\bar{y}_o$, mm³
①	2170	36174	241.2	8725169
②	37500	37500	125	4687500
		73674		13412669

$$I_1 = n_1 \bar{I}_x + n_1 A_1 d_1^2 = (16.67)(538000) + (36174)(241.2 - 182.1)^2 = 135.32 \times 10^6\ \text{mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^2 + n_2 A_2 d_2^2 = \frac{1}{12}(150)(250)^3 + (37500)(182.1 - 125)^2 = 317.58 \times 10^6\ \text{mm}^4$$

$$I = I_1 + I_2 = 452.9 \times 10^6\ \text{mm}^4.$$

$M = 50$ kNm $\qquad \sigma = -\dfrac{nMy}{I}$

(a) Wood: $n = 1$, $y = -8.433$ in.

$$\sigma_w = -\frac{(50 \times 10^3)(-182.1 \times 10^{-3})}{452.9 \times 10^{-6}} = 20.1\ \text{MPa} \qquad \sigma_w = 20.1\ \text{MPa}$$

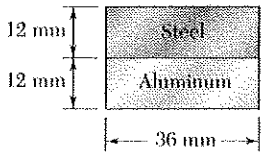
(b) Steel: $n = 16.67$ $\quad y = 250 + 5.6 - 182.1 = 73.5$ mm

$$\sigma_s = -\frac{(16.67)(50 \times 10^3)(73.5 \times 10^{-3})}{452.9 \times 10^{-6}} = -135.3\ \text{MPa} \qquad \sigma_s = -135.3\ \text{MPa}$$

Problem 4.43

4.43 and 4.44 For the composite bar indicated, determine the radius of curvature caused by the couple of moment 200 N · m.

4.43 Bar of Prob. 4.39



See solution to Problem 4.39 for calculation of I.

$I = 67.392 \times 10^{-9} m^4$ $\quad E_a = 70 \times 10^9$ Pa

$M = 200$ N·m

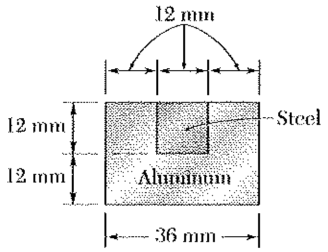
$$\frac{1}{\rho} = \frac{M}{EI} = \frac{200}{(70\times10^9)(67.392\times10^{-9})} = 42.396\times10^{-3}\ m^{-1}$$

$\rho = 23.6$ m ◄

Problem 4.44

4.43 and 4.44 For the composite bar indicated, determine the radius of curvature caused by the couple of moment 200 N · m.

4.44 Bar of Prob. 4.40



See solution to Problem 4.40 for calculation of I.

$I = 52.704 \times 10^{-9} m^4$ $\quad E_a = 70 \times 10^6$ Pa

$M = 200$ N·m

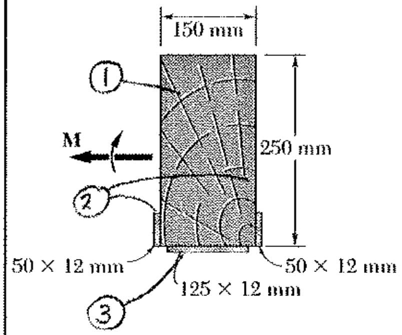
$$\frac{1}{\rho} = \frac{M}{EI} = \frac{200}{(70\times10^9)(52.704\times10^{-9})} = 54.211\ m^{-1}$$

$\rho = 18.45m$ ◄

Problem 4.45

4.45 and *4.46* For the composite beam indicated, determine the radius of curvature caused by the couple of moment 50 kN · m.

4.45 Beam of Prob. 4.41



See solution to Problem 4.41 for calculation of I.

$I = 491.53\times10^{-6} m^4$ $\quad E_w = 12 \times 10^9$ Pa

$M = 50\times10^3$ Nm

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{50\times10^3}{(12\times10^9)(491.53\times10^{-6})} = 8.4769\times10^{-3} m^{-1}$$

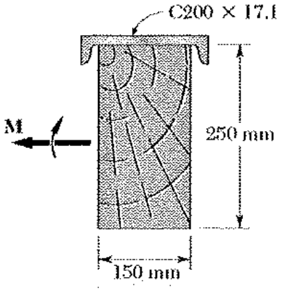
$\rho = 117.96 m = 118 m$ ◄

Problem 4.46

4.45* and *4.46 For the composite beam indicated, determine the radius of curvature caused by the couple of moment 50 kN · m.

4.45 Bar of Prob. 4.41.

4.46 Bar of Prob. 4.42.



See solution to Problem 4.42 for calculation of I.

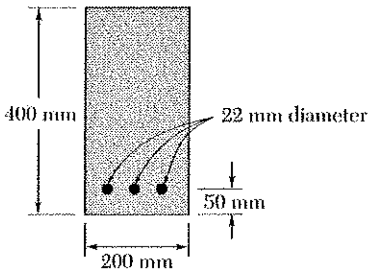
$I = 452.9\times10^{6}$ mm 4 $\quad E_w = 12$ GPa.

$M = 50$ kNm

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{50\times10^3}{(12\times10^9)(452.9\times10^{-6})} = 0.0092\ m^{-1}$$

$\rho = 108.7 m$ ◄

Problem 4.47

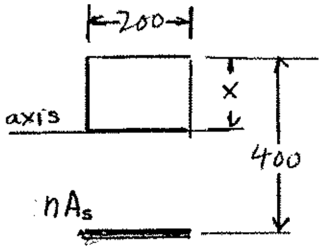


4.47 A concrete beam is reinforced by three steel rods placed as shown. The modulus of elasticity is 20 GPa for the concrete and 200 GPa for the steel. Using an allowable stress of 9.45 MPa for the concrete and 140 MPa for the steel, determine the largest allowable positive bending moment in the beam.

$$n = \frac{E_s}{E_c} = \frac{200\times10^9}{20\times10^9} = 10$$

$$A_s = 3\,\tfrac{\pi}{4}d^2 = 3\left(\tfrac{\pi}{4}\right)(22)^2 = 1140.4\ mm^2$$

$$nA_s = 11404\ mm^2$$



Locate neutral axis.

$$200x\frac{x}{2} - (11404)(400-x) = 0$$

$$100x^2 + 11404x - 4561600 = 0$$

Solve for x
$$x = \frac{-11404 + \left[11404^2 + (4)(100)(4561600)\right]^{1/2}}{2(100)} = 164\ mm$$

$$400 - x = 236\ mm$$

$$I = \tfrac{1}{3}200x^3 + nA_s(400-x)^2 = \tfrac{1}{3}(200)(164)^3 + (11404)(236)^2$$

$$= 296.75\times10^6\ mm^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \quad \therefore \quad M = \frac{\sigma I}{ny}$$

Concrete: $n = 1.0$, $|y| = 164mm$, $|\sigma| = 9.45\,MPa$

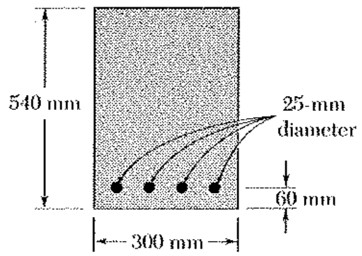
$$M = \frac{(9.45\times10^6)(296.75\times10^{-6})}{(1.0)(0.164)} = 17.1\ kNm$$

Steel: $n = 10$, $|y| = 236mm$, $\sigma = 140\,MPa$

$$M = \frac{(140000\times10^3)(296.75\times10^{-6})}{(10)(0.236)} = 17.6\ kNm$$

Choose the smaller value $M = 17.1\ kNm$ ◄

Problem 4.48

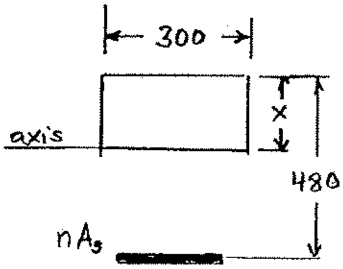


4.48 The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN · m. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.

$$n = \frac{E_s}{E_c} = \frac{200 \text{ GPa}}{25 \text{ GPa}} = 8.0$$

$$A_s = 4 \cdot \frac{\pi}{4} d^2 = (4)(\tfrac{\pi}{4})(25)^2 = 1.9635 \times 10^3 \text{ mm}^2$$

$$nA_s = 15.708 \times 10^3 \text{ mm}^2$$



Locate the neutral axis.

$$300\, x \frac{x}{2} - (15.708 \times 10^3)(480 - x) = 0$$

$$150 x^2 + 15.708 \times 10^3 x - 7.5398 \times 10^6 = 0$$

Solve for x.

$$x = \frac{-15.708 \times 10^3 + \sqrt{(15.708 \times 10^3)^2 + (4)(150)(7.5398 \times 10^6}}{(2)(150)}$$

$$x = 177.87 \text{ mm}, \qquad 480 - x = 302.13 \text{ mm}$$

$$I = \frac{1}{3} 300\, x^3 + (15.708 \times 10^3)(480 - x)^2$$

$$= \frac{1}{3}(300)(177.87)^3 + (15.708 \times 10^3)(302.13)^2$$

$$= 1.9966 \times 10^9 \text{ mm}^4 = 1.9966 \times 10^{-3} \text{ m}^4$$

$$\sigma = -\frac{nMy}{I}$$

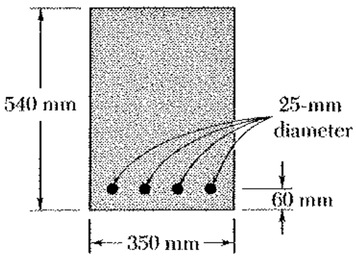
(a) Steel: $y = -302.45 \text{ mm} = -0.30245 \text{ m}$

$$\sigma = -\frac{(8.0)(175 \times 10^3)(-0.30245)}{1.9966 \times 10^{-3}} = 212 \times 10^6 \text{ Pa} = 212 \text{ MPa}$$ ◄

(b) Concrete: $y = 177.87 \text{ mm} = 0.17787 \text{ m}$

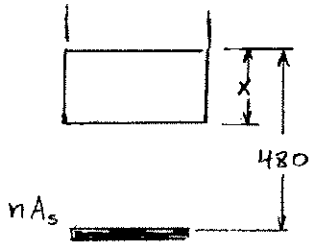
$$\sigma = -\frac{(1.0)(175 \times 10^3)(0.17787)}{1.9966 \times 10^{-3}} = -15.59 \times 10^6 \text{ Pa} = -15.59 \text{ MPa}$$ ◄

Problem 4.49



4.49 Solve Prob. 4.48, assuming that the 300-mm width is increased to 350 mm.

4.48 The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN · m. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.



$$n = \frac{E_s}{E_c} = \frac{200 \text{ GPa}}{25 \text{ GPa}} = 8.0$$

$$A_s = 4\frac{\pi}{4}d^2 = (4)(\tfrac{\pi}{4})(25)^2 = 1.9635\times10^3 \text{ mm}^2$$

$$nA_s = 15.708\times10^3 \text{ mm}^2$$

Locate the neutral axis

$$350\, x \frac{x}{2} - (15.708\times10^3)(480 - x) = 0$$

$$175x^2 + 15.708\times10^3 x - 7.5398\times10^6 = 0$$

Solve for x.
$$x = \frac{-15.708\times10^3 + \sqrt{(15.708\times10^3)^2 + (4)(175)(7.5398\times10^6)}}{(2)(175)}$$

$x = 167.48\text{ mm}$, $480 - x = 312.52 \text{ mm}$

$$I = \frac{1}{3}(350)x^3 + (15.708\times10^3)(480-x)^2$$

$$= \frac{1}{3}(350)(167.48)^3 + (15.708\times10^3)(312.52)^2$$

$$= 2.0823\times10^9 \text{ mm}^4 = 2.0823\times10^{-3} \text{ m}^4$$

$$\sigma = -\frac{nMy}{I}$$

(a) Steel: $y = -312.52 \text{ mm} = -0.31252 \text{ m}$

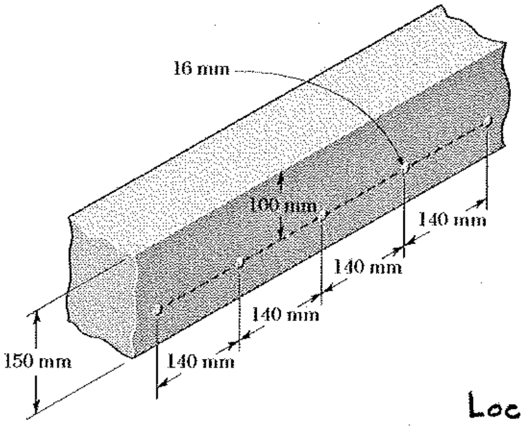
$$\sigma = -\frac{(8.0)(175\times10^3)(-0.31252)}{2.0823\times10^{-3}} = 210\times10^6 \text{ Pa} = 210 \text{ MPa} \blacktriangleleft$$

(b) Concrete: $y = 167.48 \text{ mm} = 0.16748 \text{ m}$

$$\sigma = -\frac{(1.0)(175\times10^3)(0.16748)}{2.0823\times10^{-3}} = -14.08\times10^6 \text{ Pa} = -14.08 \text{MPa} \blacktriangleleft$$

Problem 4.50

4.50 A concrete slab is reinforced by 16-mm-diameter rods placed on 140-mm centers as shown. The modulus of elasticity is 20 GPa for the concrete and 200 GPa for the steel. Using an allowable stress of 9 MPa for the concrete and 140 MPa for the steel, determine the largest bending moment per foot of width that can be safely applied to the slab.



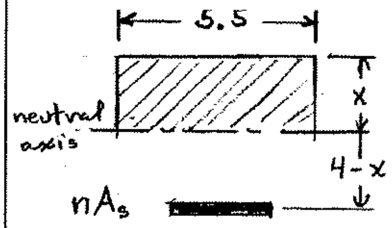
$$n = \frac{E_s}{E_c} = \frac{200\times10^9}{20\times10^9} = 10$$

Consider a section 140 mm wide.

$$A_s = \frac{\pi}{4}d_s^2 = \frac{\pi}{4}(16)^2 = 201 \ mm^2$$

$$nA_s = 2010 \ mm^2$$

Locate the neutral axis.



$$140x\frac{x}{2} - (100-x)(2010) = 0$$

$$70x^2 + 2010 \ x - 201000 = 0$$

Solving for x

$x = 41.1$ mm $100 - x = 58.9$ mm

$$I = \frac{1}{3}(140)x^3 + (2010)(100-x)^2$$
$$= \frac{1}{3}(140)(41.1)^3 + (2010)(58.9)^2$$
$$= 10.213\times10^6 \ mm^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \left|\frac{I\sigma}{ny}\right|$$

Concrete: $n = 1$, $y = 41.1$ mm, $\sigma = 9$ MPa.

$$M = \frac{(10.213\times10^{-6})(9\times10^6)}{(1.0)(0.0411)} = 2236 \ Nm$$

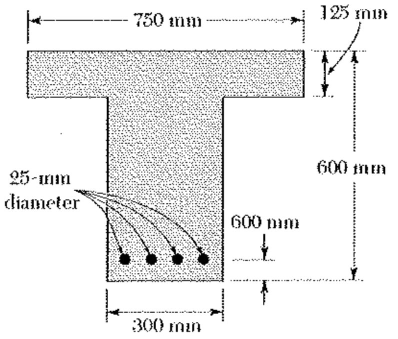
Steel: $n = 10$, $y = 58.9$ mm, $\sigma = 140$ MPa;

$$M = \frac{(10.213\times10^{-6})(140\times10^6)}{(10)(0.0589)} = 2428 \ Nm$$

Choose the smaller value as the allowable moment for a 140 mm width.

$$M = 2236 \ Nm = 2.24 \ kNm$$

Problem 4.51

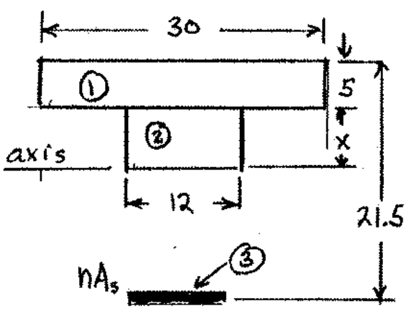


4.51 Knowing that the bending moment in the reinforced concrete beam is +200 kN · m and that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.

$n = \frac{E_s}{E_c} = \frac{200\times10^9}{25\times10^9} = 8$

$A_s = 4\frac{\pi}{4}d^2 = 4\left(\frac{\pi}{4}\right)(25)^2 = 1963.5\ mm^2.$

$nA_s = 15708\ mm^2.$



Locate the neutral axis

$(750)(125)(x+62.5) + 300x\,x/2$
$- (15708)(415-x) = 0$

$93750x + 5859375 + 150x^2 - 6518820 + 15708x = 0$

$150x^2 + 109458x - 659445 = 0$

Solve for x $x = \frac{-109458 + [(109458)^2 + (4)(150)(659445)]^{1/2}}{2(150)} = 6\ mm$

$415 - x = 409\ mm$

$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(750)(125)^3 + (750)(125)(68.125)^2 = 557.166\times10^6\ mm^4$

$I_2 = \frac{1}{3}b_2x^3 = \frac{1}{3}(12)(6)^3 = 864\ mm^4.$

$I_3 = nA_sd_3^2 = (15708)(409)^2 = 2627.65\times10^6\ mm^4$

$I = I_1 + I_2 + I_3 = 3.1848\times10^9\ mm^4$

$\sigma = -\frac{nMy}{I}$ where $M = 200\ kNm$

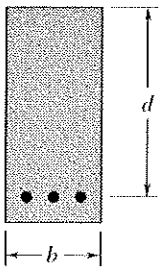
(a) Steel $n = 8.0$, $y = -409\ mm$

$\sigma = -\frac{(8.0)(200\times10^3)(409\times10^{-3})}{3.1848\times10^{-3}} = 205.5\ MPa.$ ◄

(b) Concrete $n = 1.0$, $y = 131\ mm$

$\sigma = -\frac{(1.0)(200\times10^3)(0.131)}{3.1848\times10^{-3}} = -8.2\ MPa.$ ◄

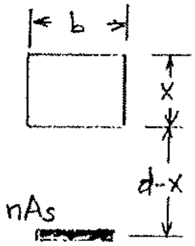
Problem 4.52



4.52 The design of a reinforced concrete bean is said to be *balanced* if the maximum stresses in the steel and concrete are equal, respectively, to the allowable stresses σ_s and σ_c. Show that to achieve a balanced design the distance x from the top of the beam to the neutral axis must be

$$x = \frac{d}{1 + \dfrac{\sigma_s E_c}{\sigma_c E_x}}$$

where E_c and E_s are the moduli of elasticity of concrete and steel, respectively, and d is the distance from the top of the beam to the reinforcing steel.



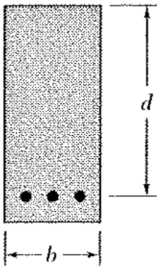
$$\sigma_s = \frac{nM(d-x)}{I} \qquad \sigma_c = \frac{Mx}{I}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{n(d-x)}{x} = n\frac{d}{x} - n$$

$$\frac{d}{x} = 1 + \frac{1}{n}\frac{\sigma_s}{\sigma_c} = 1 + \frac{E_c\sigma_s}{E_s\sigma_c}$$

$$x = \frac{d}{1 + \dfrac{E_c\sigma_s}{E_s\sigma_c}}$$

Problem 4.53



4.53 For the concrete beam shown, the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel. Knowing that $b = 200$ mm and $d = 450$ mm, and using an allowable stress of 12.5 MPa for the concrete and 140 MPa for the steel, determine (*a*) the required area A_s of the steel reinforcement if the beam is to be balanced, (*b*) the largest allowable bending moment. (See Prob. 4.52 for definition of a balanced beam.)

$$n = \frac{E_s}{E_c} = \frac{200 \times 10^9}{25 \times 10^9} = 8.0$$

$$\sigma_s = \frac{nM(d-x)}{I} \qquad \sigma_c = \frac{Mx}{I}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{n(d-x)}{x} = n\frac{d}{x} - n$$

$$\frac{d}{x} = 1 + \frac{1}{n}\frac{\sigma_s}{\sigma_c} = 1 + \frac{1}{8.0} \cdot \frac{140 \times 10^6}{12.5 \times 10^6} = 2.40$$

$$x = 0.41667\,d = (0.41667)(450) = 187.5 \text{ mm}$$

Locate neutral axis.

$$bx\frac{x}{2} - nA_s(d-x)$$

(a) $$A_s = \frac{bx^2}{2n(d-x)} = \frac{(200)(187.5)^2}{(2)(8.0)(262.5)} = 1674 \text{ mm}^2$$ (a) $A_s = 1674 \text{ mm}^2$ ◄

$$I = \frac{1}{3}bx^3 + nA_s(d-x)^2 = \frac{1}{3}(200)(187.5)^3 + (8.0)(1674)(262.5)^2$$

$$= 1.3623 \times 10^9 \text{ mm}^4 = 1.3623 \times 10^{-3} \text{ m}^4$$

$$\sigma = \frac{nMy}{I} \qquad M = \frac{I\sigma}{ny}$$

Concrete: $n = 1.0 \qquad y = 187.5 \text{ mm} = 0.1875 \text{ m} \qquad \sigma = 12.5 \times 10^6 \text{ Pa}$

$$M = \frac{(1.3623 \times 10^{-3})(12.5 \times 10^6)}{(1.0)(0.1875)} = 90.8 \times 10^3 \text{ N-m}$$

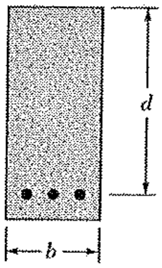
Steel: $n = 8.0 \qquad y = 262.5 \text{ mm} = 0.2625 \text{ m} \qquad \sigma = 140 \times 10^6 \text{ Pa}$

$$M = \frac{(1.3623 \times 10^{-3}(140 \times 10^6)}{(8.0)(0.2625)} = 90.8 \times 10^3 \text{ N-m}$$

Note that both values are the same for balanced design.

(b) $M = 90.8$ kN·m ◄

Problem 4.54



4.54 For the concrete beam shown, the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel. Knowing that $b = 200$ mm and $d = 450$ mm and using an allowable stress of 12.5 MPa for the concrete and 140 MPa for the steel, determine *(a)* the required area A_s of the steel reinforcement if the beam is to be balanced, *(b)* the largest allowable bending moment. (See Prob. 4.52 for definition of a balanced beam.)

$$n = \frac{E_s}{E_c} = \frac{200 \times 10^9}{25 \times 10^9} = 8.0$$

$$\sigma_s = \frac{nM(d-x)}{I} \qquad \sigma_c = \frac{Mx}{I}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{n(d-x)}{x} = n\frac{d}{x} - n$$

$$\frac{d}{x} = 1 + \frac{1}{n}\frac{\sigma_s}{\sigma_c} = 1 + \frac{1}{8.0}\cdot\frac{140 \times 10^6}{12.5 \times 10^6} = 2.40$$

$$x = 0.41667\,d = (0.41667)(450) = 187.5 \text{ mm}$$

Locate neutral axis

$$b x \frac{x}{2} - n A_s (d-x)$$

(a) $$A_s = \frac{bx^2}{2n(d-x)} = \frac{(200)(187.5)^2}{(2)(8.0)(262.5)} = 1674 \text{ mm}^2$$ (a) $A_s = 1674 \text{ mm}^2$ ◄

$$I = \tfrac{1}{3} b x^3 + n A_s (d-x)^2 = \tfrac{1}{3}(200)(187.5)^3 + (8.0)(1674)(262.5)^2$$

$$= 1.3623 \times 10^9 \text{ mm}^4 = 1.3623 \times 10^{-3} \text{ m}^4$$

$$\sigma = \frac{nMy}{I} \qquad M = \frac{I\sigma}{ny}$$

Concrete: $n = 1.0$ $\quad y = 187.5 \text{ mm} = 0.1875 \text{ m}$ $\quad \sigma = 12.5 \times 10^6$ Pa

$$M = \frac{(1.3623 \times 10^{-3})(12.5 \times 10^6)}{(1.0)(0.1875)} = 90.8 \times 10^3 \text{ N-m}$$

Steel: $n = 8.0$ $\quad y = 262.5 \text{ mm} = 0.2625 \text{ m}$ $\quad \sigma = 140 \times 10^6$ Pa

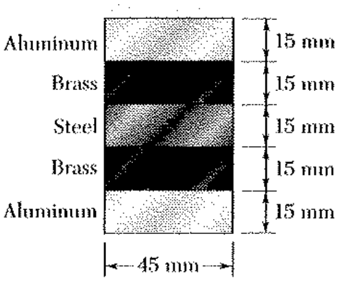
$$M = \frac{(1.3623 \times 10^{-3}(140 \times 10^6))}{(8.0)(0.2625)} = 90.8 \times 10^3 \text{ N-m}$$

Note that both values are the same for balanced design.

(b) $M = 90.8$ kN-m ◄

Problem 4.55

4.55 and 4.56 Five metal strips, each of 15 × 45-mm cross section, are bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel, 105 GPa for the brass, and 70 GPa for the aluminum. Knowing that the beam is bent about a horizontal axis by couple of moment 1400 N · m, determine (*a*) the maximum stress in each of the three metals, (*b*) the radius of curvature of the composite beam.

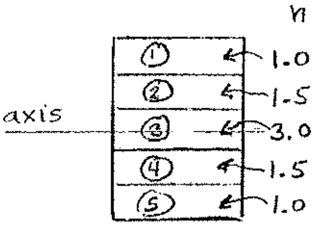


Use aluminum as the reference material.

$$n = \frac{E_s}{E_a} = \frac{210}{70} = 3.0 \text{ in steel.}$$

$$n = \frac{E_b}{E_a} = \frac{105}{70} = 1.5 \text{ in brass.}$$

$$n = 1.0 \text{ in aluminum.}$$



For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2$$

$$= \frac{1.0}{12}(45)(15)^3 + (1.0)(45)(15)(30)^2 = 620.16 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.5}{12}(45)(15)^3 + (1.5)(45)(15)(15)^2 = 246.80 \times 10^3 \text{ mm}^4$$

$$I_3 = \frac{n_3}{12} b_3 h_3^3 = \frac{3}{12}(45)(15)^3 = 37.97 \times 10^3 \text{ mm}^4$$

$$I_4 = I_2 = 246.80 \times 10^3 \text{ mm}^4 \qquad I_5 = I_1 = 620.16 \times 10^3 \text{ mm}^4$$

$$I = \sum_1^5 I_i = 1.77189 \times 10^6 \text{ mm}^4 = 1.77189 \times 10^{-6} \text{ m}^4$$

(a) Aluminum: $y = 37.5 \text{ mm} = 0.0375 \text{ m} \qquad n = 1.0$

$$\sigma = \frac{nMy}{I} = \frac{(1.0)(1400)(0.0375)}{1.77189 \times 10^{-6}} = 29.6 \times 10^6 \text{ Pa} \qquad \sigma_a = 29.6 \text{ MPa} \blacktriangleleft$$

Brass: $y = 22.5 \text{ mm} = 0.0225 \text{ m} \qquad n = 1.5$

$$\sigma = \frac{nMy}{I} = \frac{(1.5)(1400)(0.0225)}{1.77189 \times 10^{-6}} = 26.7 \times 10^6 \text{ Pa} \qquad \sigma_b = 26.7 \text{ MPa} \blacktriangleleft$$

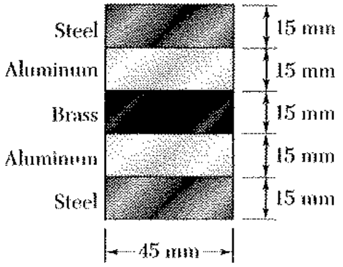
Steel: $y = 7.5 \text{ mm} = 0.0075 \text{ m} \qquad n = 3.0$

$$\sigma = \frac{nMy}{I} = \frac{(3.0)(1400)(0.0075)}{1.77189 \times 10^{-6}} = 17.78 \times 10^6 \text{ Pa} \qquad \sigma_s = 17.78 \text{ MPa} \blacktriangleleft$$

(b) $$\frac{1}{\rho} = \frac{M}{E_a I} = \frac{1400}{(70 \times 10^9)(1.77189 \times 10^{-6})} = 11.287 \times 10^{-3} \text{ m}^{-1}$$

$$\rho = 88.6 \text{ m} \blacktriangleleft$$

Problem 4.56



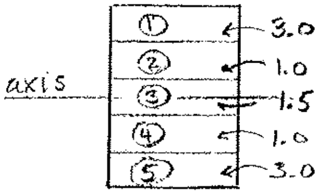
4.55 and 4.56 Five metal strips, each of 15 × 45-mm cross section, are bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel, 105 GPa for the brass, and 70 GPa for the aluminum. Knowing that the beam is bent about a horizontal axis by couple of moment 1400 N · m, determine (*a*) the maximum stress in each of the three metals, (*b*) the radius of curvature of the composite beam.

Use aluminum as the reference material.

$n = \frac{E_s}{E_a} = \frac{210}{70} = 3.0$ in steel.

$n = \frac{E_b}{E_a} = \frac{210}{105} = 1.5$ in brass.

$n = 1.0$ in aluminum



For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2$$

$$= \frac{3.0}{12}(45)(15)^3 + (3.0)(45)(15)(30)^2 = 1.860469 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.0}{12}(45)(15)^3 + (1.0)(45)(15)(15)^2 = 164.53 \times 10^3 \text{ mm}^4$$

$$I_3 = \frac{n_3}{12} b_3 h_3^3 = \frac{1.5}{12}(45)(15)^3 = 18.98 \times 10^3 \text{ mm}^4$$

$I_4 = I_2 = 164.53 \times 10^3 \text{ mm}^4$ $\qquad$ $I_5 = I_1 = 1.860469 \times 10^6 \text{ mm}^4$

$$I = \sum_1^5 I_i = 4.0690 \times 10^6 \text{ mm}^4 = 4.0690 \times 10^{-6} \text{ m}^4$$

(a) Steel: $y = 37.5 \text{ mm} = 0.0375 \text{ m}$ $\qquad$ $n = 3.0$

$$\sigma = \frac{nMy}{I} = \frac{(3.0)(1400)(0.0375)}{4.0690 \times 10^{-6}} = 38.7 \times 10^6 \text{ Pa} \qquad \sigma_s = 38.7 \text{ MPa}$$

Aluminum: $y = 22.5 \text{ mm} = 0.0225 \text{ m}$ $\qquad$ $n = 1.0$

$$\sigma = \frac{nMy}{I} = \frac{(1.0)(1400)(0.0225)}{4.0690 \times 10^{-6}} = 7.74 \times 10^6 \text{ Pa} \qquad \sigma_a = 7.74 \text{ MPa}$$

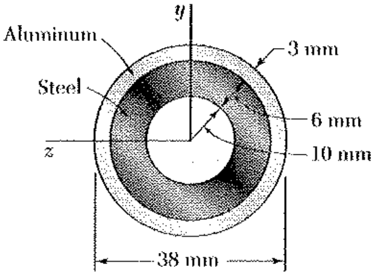
Brass: $y = 7.5 \text{ mm} = 0.0075 \text{ m}$ $\qquad$ $n = 1.5$

$$\sigma = \frac{nMy}{I} = \frac{(1.5)(1400)(0.0075)}{4.0690 \times 10^{-6}} = 3.87 \times 10^6 \text{ Pa} \qquad \sigma_b = 3.87 \text{ MPa}$$

(b) $$\frac{1}{\rho} = \frac{M}{E_a I} = \frac{1400}{(70 \times 10^9)(4.0690 \times 10^{-6})} = 4.9152 \times 10^{-3} \text{ m}^{-1}$$

$$\rho = 203 \text{ m}$$

Problem 4.57



4.57 A steel pipe and an aluminum pipe are securely bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel and 70 GPa for the aluminum. Knowing that the composite beam is bent by couple of moment 500 N · m, determine the maximum stress (*a*) in the aluminum, (*b*) in the steel.

Use aluminum as the reference material

$n = 1.0$ in aluminum

$n = \frac{E_s}{E_a} = \frac{210}{70} = 3.0$ in steel

Steel: $I_1 = n_1 \frac{\pi}{4}(r_o^4 - r_i^4) = (3.0)\frac{\pi}{4}(16^4 - 10^4) = 130.85 \times 10^3 \text{ mm}^4$

Aluminum: $I_2 = n_2 \frac{\pi}{4}(r_o^4 - r_i^4) = (1.0)\frac{\pi}{4}(19^4 - 16^4) = 50.88 \times 10^3 \text{ mm}^4$

$I = I_1 + I_2 = 181.73 \times 10^3 \text{ mm}^4 = 181.73 \times 10^{-9} \text{ m}^4$

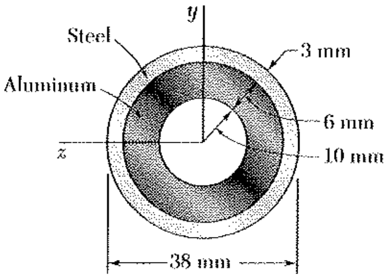
(a) Aluminum: $c = 19 \text{ mm} = 0.019 \text{ m}$

$$\sigma = \frac{nMc}{I} = \frac{(1.0)(500)(0.019)}{181.73 \times 10^{-9}} = 52.3 \times 10^6 \text{ Pa} = 52.3 \text{ MPa}$$ ◄

(b) Steel: $c = 16 \text{ mm} = 0.016 \text{ m}$

$$\frac{nMc}{I} = \frac{(3.0)(500)(0.016)}{181.73 \times 10^{-9}} = 132.1 \times 10^6 \text{ Pa} = 132.1 \text{ MPa}$$ ◄

Problem 4.58



4.58 Solve Prob. 4.57, assuming that the 6-mm-thick inner pipe is made of aluminum and that the 3-mm-thick outer pipe is made of steel.

4.57 A steel pipe and an aluminum pipe are securely bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel and 70 GPa for the aluminum. Knowing that the composite beam is bent by couple of moment 500 N · m, determine the maximum stress (*a*) in the aluminum, (*b*) in the steel.

Use aluminum as the reference material

$n = 1.0$ in aluminum

$n = \frac{E_s}{E_a} = \frac{210}{70} = 3.0$ in steel.

Steel: $I_1 = n_1 \frac{\pi}{4}(r_o^4 - r_i^4) = (3.0)\frac{\pi}{4}(19^4 - 16^4) = 152.65 \times 10^3 \text{ mm}^4$

Aluminum: $I_2 = n_2 \frac{\pi}{4}(r_o^4 - r_i^4) = (1.0)\frac{\pi}{4}(16^4 - 10^4) = 43.62 \times 10^3 \text{ mm}^4$

$I = I_1 + I_2 = 196.27 \times 10^3 \text{ mm}^4 = 196.27 \times 10^{-9} \text{ m}^4$

(a) Aluminum: $c = 16 \text{ mm} = 0.016 \text{ m}$

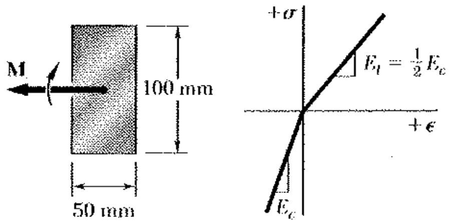
$\sigma = \frac{n M c}{I} = \frac{(1.0)(500)(0.016)}{196.27 \times 10^{-9}} = 40.8 \times 10^6 \text{ Pa} = 40.8 \text{ MPa}$ ◀

(b) Steel: $c = 19 \text{ mm} = 0.019 \text{ m}$

$\sigma = \frac{n M c}{I} = \frac{(3.0)(500)(0.019)}{196.27 \times 10^{-9}} = 145.2 \times 10^6 \text{ Pa} = 145.2 \text{ MPa}$ ◀

Problem 4.59

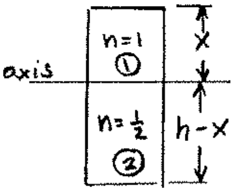
4.59 The rectangular beam shown is made of a plastic for which the value of the modulus of elasticity in tension is one-half of its value in compression. For a bending moment $M = 600$ N · m, determine the maximum (*a*) tensile stress, (*b*) compressive stress.



$n = \frac{1}{2}$ on the tension side of neutral axis.

$n = 1$ on the compression side.

Locate neutral axis.



$$n_1 b x \frac{x}{2} - n_2 b(h-x)\frac{h-x}{2} = 0$$

$$\frac{1}{2}bx^2 - \frac{1}{4}b(h-x)^2 = 0$$

$$x^2 = \frac{1}{2}(h-x)^2 \qquad x = \frac{1}{\sqrt{2}}(h-x)$$

$$x = \frac{1}{\sqrt{2}+1}h = 0.41421\ h = 41.421 \text{ mm}$$

$$h - x = 58.579 \text{ mm}$$

$$I_1 = n_1 \tfrac{1}{3} b x^3 = (1)(\tfrac{1}{3})(50)(41.421)^3 = 1.1844 \times 10^6 \text{ mm}^4$$

$$I_2 = n_2 \tfrac{1}{3} b(h-x)^3 = (\tfrac{1}{2})(\tfrac{1}{3})(50)(58.579)^3 = 1.6751 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 = 2.8595 \times 10^6 \text{ mm}^4 = 2.8595 \times 10^{-6} \text{ m}^4$$

(a) <u>tensile stress</u>: $n = \frac{1}{2}$, $y = -58.579$ mm $= -0.058579$ m

$$\sigma = -\frac{nMy}{I} = -\frac{(0.5)(600)(-0.058579)}{2.8595\times10^{-6}} = 6.15 \times 10^6 \text{ Pa}$$

$$\sigma_T = 6.15 \text{ MPa}$$

(b) <u>compressive stress</u>: $n = 1$, $y = 41.421$ mm $= 0.041421$ m

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(600)(0.041421)}{2.8595\times10^{-6}} = -8.69 \times 10^6 \text{ Pa}$$

$$\sigma_c = -8.69 \text{ MPa}$$

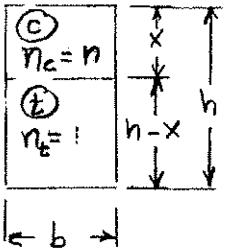
Problem 4.60

***4.60** A rectangular beam is made of material for which the modulus of elasticity is E_t in tension and E_c in compression. Show that the curvature of the beam in pure bending is

$$\frac{1}{\rho} = \frac{M}{E_r I}$$

where

$$E_r = \frac{4E_t E_c}{\left(\sqrt{E_t} + \sqrt{E_c}\right)^2}$$



Use E_t as the reference modulus.

Then $E_c = n E_t$

Locate neutral axis.

$$n b x \frac{x}{2} - b(h-x)\frac{h-x}{2} = 0$$

$$n x^2 - (h-x)^2 = 0 \qquad \sqrt{n}\, x = (h-x)$$

$$x = \frac{h}{\sqrt{n}+1} \qquad h-x = \frac{\sqrt{n}\, h}{\sqrt{n}+1}$$

$$I_{trans} = \frac{n}{3} b x^3 + \frac{1}{3} b (h-x)^3 = \left[\frac{n}{3}\cdot\left(\frac{1}{\sqrt{n}+1}\right)^3 + \left(\frac{\sqrt{n}}{\sqrt{n}+1}\right)^3\right] b h^3$$

$$= \frac{1}{3}\,\frac{n + n^{3/2}}{(\sqrt{n}+1)^3}\, b h^3 = \frac{1}{3}\,\frac{n(1+\sqrt{n})}{(\sqrt{n}+1)^3}\, b h^3 = \frac{1}{3}\cdot\frac{n}{(\sqrt{n}+1)^2}\, b h^3$$

$$\frac{1}{\rho} = \frac{M}{E_t I_{trans}} = \frac{M}{E_r I} \qquad \text{where } I = \frac{1}{12} b h^3$$

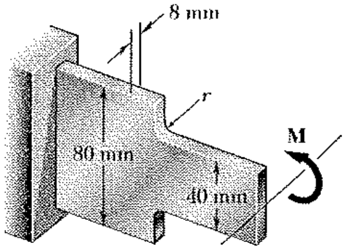
$$E_r I = E_t I_{trans}$$

$$E_r = \frac{E_t I_{trans}}{I} = \frac{12}{bh^3}\cdot E_t \cdot \frac{n}{3(\sqrt{n}+1)^2}\, bh^3$$

$$= \frac{4 E_t E_c / E_t}{\left(\sqrt{E_c/E_t} + 1\right)^2} = \frac{4 E_t E_c}{\left(\sqrt{E_c} + \sqrt{E_t}\right)^2}$$

Problem 4.61

4.61 **Knowing that the allowable stress for the beam shown is 90 MPa, determine (*a*) the allowable bending moment *M* when the radius *r* of the fillets is (*a*) 8 mm, (*b*) 12 mm.**



$I = \frac{1}{12}bh^3 = \frac{1}{12}(8)(40)^3 = 42.667 \times 10^3\ mm^4 = 42.667 \times 10^{-9}\ m^4$

$c = 20\ mm = 0.020\ m$

$\frac{D}{d} = \frac{80\ mm}{40\ mm} = 2.00$

(a) $\frac{r}{d} = \frac{8\ mm}{40\ mm} = 0.2$ From Fig. 4.31 $K = 1.50$

$\sigma_{max} = K\frac{Mc}{I} \therefore M = \frac{\sigma_{max} I}{Kc} = \frac{(90 \times 10^6)(42.667 \times 10^{-9})}{(1.50)(0.020)}$

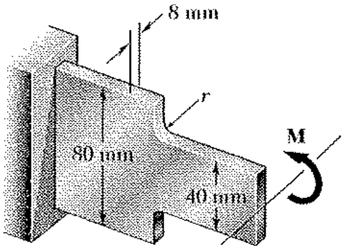
$= 128\ N \cdot m$ ◄

(b) $\frac{r}{d} = \frac{12\ mm}{40\ mm} = 0.3$ From Fig 4.31 $K = 1.35$

$M = \frac{(90 \times 10^6)(42.667 \times 10^{-9})}{(1.35)(0.020)} = 142\ N \cdot m$ ◄

Problem 4.62

4.62 **Knowing that *M* = 250 N ·m, determine the maximum stress in the beam shown when the radius *r* of the fillets is (*a*) 4 mm, (*b*) 8 mm.**



$I = \frac{1}{12}bh^3 = \frac{1}{12}(8)(40)^3 = 42.667 \times 10^3\ mm^4 = 42.667 \times 10^{-9}\ m^4$

$c = 20\ mm = 0.020\ m$

$\frac{D}{d} = \frac{80\ mm}{40\ mm} = 2.00$

(a) $\frac{r}{d} = \frac{4\ mm}{40\ mm} = 0.10$ From Fig. 4.31 $K = 1.87$

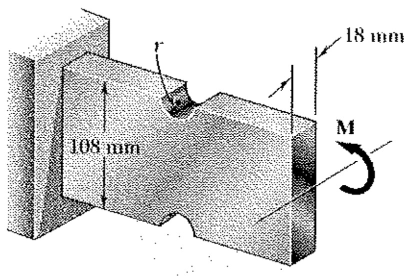
$\sigma_{max} = K\frac{Mc}{I} = \frac{(1.87)(250)(0.020)}{42.667 \times 10^{-9}} = 219 \times 10^6\ Pa = 219\ MPa$ ◄

(b) $\frac{r}{d} = \frac{8\ mm}{40\ mm} = 0.20$ From Fig. 4.31 $K = 1.50$

$\sigma_{max} = K\frac{Mc}{I} = \frac{(1.50)(250)(0.020)}{42.667 \times 10^{-9}} = 176 \times 10^6\ Pa = 176\ MPa$ ◄

Problem 4.63

4.63 Semicircular grooves of radius *r* must be milled as shown in the sides of a steel member. Knowing that *M* = 450 N · m, determine the maximum stress in the member when the radius *r* of the semicircular grooves is (*a*) *r* = 9 mm, (*b*) *r* = 18 mm.



(a) $d = D - 2r = 108 - (2)(9) = 90 \text{ mm}$

$\frac{D}{d} = \frac{108}{90} = 1.20 \qquad \frac{r}{d} = \frac{9}{90} = 0.1$

From Fig. 4.32, K = 2.07

$I = \frac{1}{12} b h^3 = \frac{1}{12}(18)(90)^3 = 1.0935 \times 10^6 \text{ mm}^4 = 1.0935 \times 10^{-6} \text{ m}^4$

$c = \frac{1}{2} d = 45 \text{ mm} = 0.045 \text{ m}$

$\sigma_{max} = \frac{KMc}{I} = \frac{(2.07)(450)(0.045)}{1.0935 \times 10^{-6}} = 38.3 \times 10^6 \text{ Pa} \qquad \sigma_{max} = 38.3 \text{ MPa}$ ◄

(b) $d = D - 2r = 108 - (2)(18) = 72 \text{mm} \qquad \frac{D}{d} = \frac{108}{72} = 1.5 \qquad \frac{r}{d} = \frac{18}{72} = 0.25$

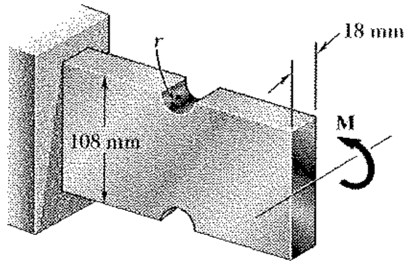
From Fig. 4.32, K = 1.61 $\qquad c = \frac{1}{2} d = 72 \text{ mm} = 0.036 \text{ m}$

$I = \frac{1}{12}(18)(72)^3 = 559.87 \times 10^3 \text{ mm}^4 = 559.87 \times 10^{-9} \text{ m}^4$

$\sigma_{max} = \frac{KMc}{I} = \frac{(1.61)(450)(0.036)}{559.87 \times 10^{-9}} = 46.6 \times 10^6 \text{ Pa} \qquad \sigma_{max} = 46.6 \text{MPa}$ ◄

Problem 4.64

4.64 Semicircular grooves of radius *r* must be milled as shown in the sides of a steel member. Using an allowable stress of 60 MPa, determine the largest bending moment that can be applied to the member when (*a*) *r* = 9 mm, (*b*) *r* = 18 mm.



(a) $d = D - 2r = 108 - (2)(9) = 90 \text{ mm}$

$\frac{D}{d} = \frac{108}{90} = 1.20 \qquad \frac{r}{d} = \frac{9}{90} = 0.1$

From Fig. 4.32, K = 2.07

$I = \frac{1}{12}(18)(90)^3 = 1.0935 \times 10^6 \text{ mm}^4$
$= 1.0935 \times 10^{-6} \text{ m}^4$

$c = \frac{1}{2} d = 45 \text{ mm} = 0.045 \text{ m}$

$\sigma = \frac{KMc}{I} \qquad M = \frac{\sigma I}{Kc} = \frac{(60 \times 10^6)(1.0935 \times 10^{-6})}{(2.07)(0.045)} = 704 \qquad M = 704 \text{ N·m}$ ◄

(b) $d = 108 - (2)(18) = 72 \text{ mm} \qquad \frac{D}{d} = \frac{108}{72} = 1.5 \qquad \frac{r}{d} = \frac{18}{72} = 0.25$

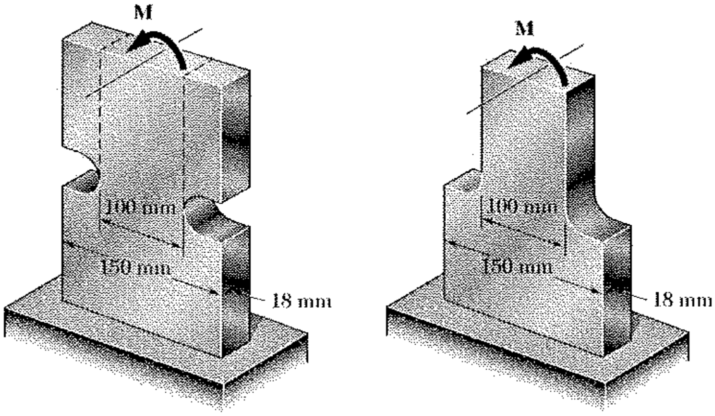
$c = \frac{1}{2} d = 36 \text{mm} = 0.036 \text{ m}$ $\qquad$ From Fig. 4.32, K = 1.61

$I = \frac{1}{12}(18)(72)^3 = 559.87 \times 10^3 \text{ mm}^4 = 559.87 \times 10^{-9} \text{ m}^4$

$M = \frac{\sigma I}{Kc} = \frac{(60 \times 10^6)(559.87 \times 10^{-9})}{(1.61)(0.036)} = 580 \qquad M = 580 \text{ N·m}$ ◄

Problem 4.65

4.65 A couple of moment $M = 2$ kN · m is to be applied to the end of a steel bar. Determine the maximum stress in the bar (*a*) if the bar is designed with grooves having semicircular portions of radius $r = 10$ mm, as shown in Fig. 4.65*a*, (*b*) if the bar is redesigned by removing the material above the grooves as shown in Fig. 4.65*b*.



(*a*) (*b*)

For both configurations

$D = 150$ mm, $d = 100$ mm

$r = 10$ mm

$\frac{D}{d} = \frac{150}{100} = 1.50$

$\frac{r}{d} = \frac{10}{100} = 0.10$

For configuration (a),
Fig 4.32 gives $K_a = 2.21$.

For configuration (b), Fig. 4.31 gives $K_b = 1.79$.

$I = \frac{1}{12} b h^3 = \frac{1}{12}(18)(100)^3 = 1.5 \times 10^6 \text{ mm}^4 = 1.5 \times 10^{-6} \text{ m}^4$

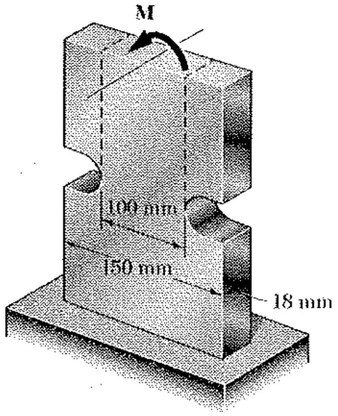
$c = \frac{1}{2} d = 50 \text{ mm} = 0.05 \text{ m}$

(a) $\sigma = \frac{KMc}{I} = \frac{(2.21)(2 \times 10^3)(0.05)}{1.5 \times 10^{-6}} = 147 \times 10^6 \text{ Pa} = 147 \text{ MPa}$ ◄

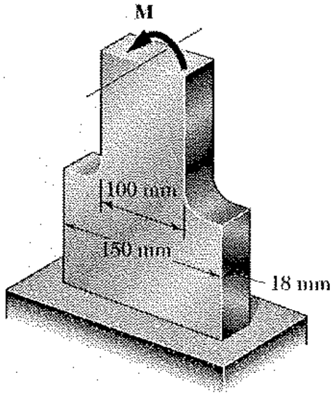
(b) $\sigma = \frac{KMc}{I} = \frac{(1.79)(2 \times 10^3)(0.05)}{1.5 \times 10^{-6}} = 119 \times 10^6 \text{ Pa} = 119 \text{ MPa}$ ◄

Problem 4.66

4.66 The allowable stress used in the design of a steel bar is 80 MPa. Determine the largest couple **M** that can be applied to the bar (*a*) if the bar is designed with grooves having semicircular portions of radius $r = 15$ mm, as shown in Fig. 4.65*a*, (*b*) if the bar is redesigned by removing the material above the grooves as shown in Fig. 4.65*b*.



(*a*)



(*b*)

For both configuations

$D = 150$ mm, $d = 100$ mm,

$r = 15$ mm.

$$\frac{D}{d} = \frac{150}{100} = 1.50$$

$$\frac{r}{d} = \frac{15}{100} = 0.15$$

For configuration (a), Fig 4.32 gives $K_a = 1.92$.

For configuration (b) Fig 4.31 gives $K_b = 1.57$.

$$I = \frac{1}{12} b h^3 = \frac{1}{12}(18)(100)^3 = 1.5 \times 10^6 \text{ mm}^4 = 1.5 \times 10^{-6} \text{ m}^4$$

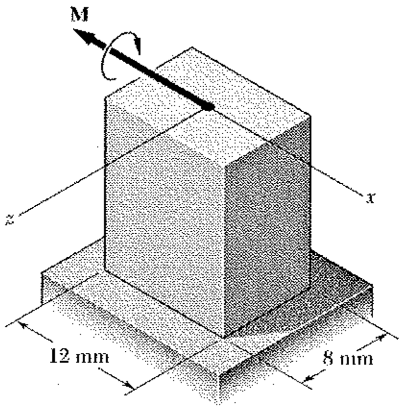
$$c = \frac{1}{2} d = 50 \text{ mm} = 0.050 \text{ m}$$

(a) $\sigma = \frac{KMc}{I} \quad \therefore \quad M = \frac{\sigma I}{Kc} = \frac{(80 \times 10^6)(1.5 \times 10^{-6})}{(1.92)(0.05)} = 1.25 \times 10^3 \text{ N·m}$

$= 1.25$ kN·m ◄

(b) $M = \frac{\sigma I}{Kc} = \frac{(80 \times 10^6)(1.5 \times 10^{-6})}{(1.57)(0.050)} = 1.53 \times 10^3 \text{ N·m} = 1.53 \text{ kN·m}$ ◄

Problem 4.67



4.67 The prismatic bar shown is made of a steel that is assumed to be elastoplastic with $\sigma_Y = 300$ MPa and is subjected to a couple **M** parallel to the *x* axis. Determine the moment *M* of the couple for which (*a*) yield first occurs, (*b*) the elastic core of the bar is 4 mm thick.

(a) $I = \frac{1}{12}bh^3 = \frac{1}{12}(12)(8)^3 - 512 \text{ mm}^4$

$= 512 \times 10^{-12} \text{ m}^4$

$c = \frac{1}{2}h = 4 \text{ mm} = 0.004 \text{ m}$

$M_Y = \frac{\sigma_Y I}{c} = \frac{(300\times10^6)(512\times10^{-12})}{0.004}$

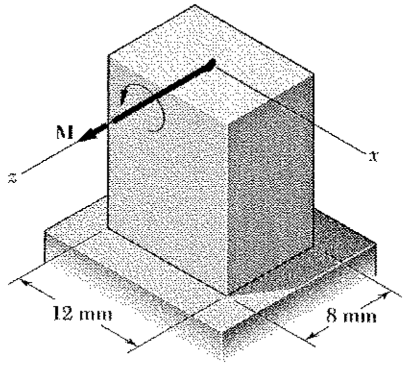
$= 38.4 \text{ N-m}$ $\qquad M_Y = 38.4 \text{ N}\cdot\text{m}$ ◄

(b) $y_Y = \frac{1}{2}(4) = 2 \text{mm} \qquad \frac{y_Y}{c} = \frac{2}{4} = 0.5$

$$M = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right] = \frac{3}{2}(38.4)\left[1 - \frac{1}{3}(0.5)^2\right] = 52.8 \text{ N}\cdot\text{m}$$

$M = 52.8 \text{ N}\cdot\text{m}$ ◄

Problem 4.68



4.68 Solve Prob. 4.67, assuming that the couple **M** is parallel to the *z* axis.

4.67 The prismatic bar shown is made of a steel that is assumed to be elastoplastic with $\sigma_Y = 300$ MPa and is subjected to a couple **M** parallel to the *x* axis. Determine the moment *M* of the couple for which (*a*) yield first occurs, (*b*) the elastic core of the bar is 4 mm thick.

(a) $I = \frac{1}{12}bh^3 = \frac{1}{12}(8)(12)^3 = 1.152\times10^3 \text{ mm}^4$

$= 1.152\times10^{-9} \text{ m}^4$

$c = \frac{1}{2}h = \quad \text{mm} = 0.006 \text{ m}$

$M_Y = \frac{\sigma_Y I}{c} = \frac{(300\times10^6)(1.152\times10^{-9})}{0.006}$

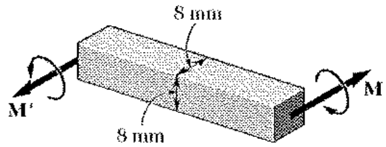
$= 57.6 \text{ N}\cdot\text{m}$ $\qquad M_Y = 57.6 \text{ N}\cdot\text{m}$ ◄

(b) $y_Y = \frac{1}{2}(4) = 2 \text{mm} \qquad \frac{y_Y}{c} = \frac{2}{6} = \frac{1}{3}$

$$M = \frac{3}{2}M_y\left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right] = \frac{3}{2}(57.6)\left[1 - \frac{1}{3}\left(\frac{1}{3}\right)^2\right] = 83.2 \text{ N}\cdot\text{m}$$

$M = 83.2 \text{ N}\cdot\text{m}$ ◄

Problem 4.69



4.69 For the steel bar of Prob. 4.70, determine the thickness of the plastic zones at the top and bottom of the bar when (*a*) $M = 30$ N.m, (*b*) $M = 35$ N.m.

4.70 A bar having the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 330$ MPa. Determine the bending moment M at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 2 mm thick.

$$I = \frac{1}{12} b h^3 = \frac{1}{12}(8)(8)^3 = 341.3 \text{ mm}^4$$

$$c = \frac{1}{2} h = 4 \text{ mm}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(330\times10^6)(341.3\times10^{-12})}{0.004} = 28.16 \text{Nm}$$

$$M_y = 28.16 \text{ Nm}.$$

$$M = \frac{3}{2} M_Y \left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right]$$

$$\frac{y_Y}{c} = \sqrt{3 - 2\frac{M}{M_Y}}$$

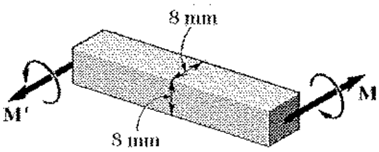
(a) $M = 30 \text{Nm}$ $\qquad \frac{y_Y}{c} = \sqrt{3 - 2\left(\frac{30}{28.16}\right)} = 0.9324$

$y_Y = (0.9324)(4) = 3.73$ mm $\qquad t_p = 4 - 3.73 = 0.27$ mm ◄

(b) $M = 35$ Nm $\qquad \frac{y_Y}{c} = \sqrt{3 - 2\left(\frac{35}{28.16}\right)} = 0.7171$

$y_Y = (0.7171)(4) = 2.87$ mm $\qquad t_p = c - y_Y = 1.13$ mm ◄

Problem 4.70



4.70 A bar having the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 330$ MPa. Determine the bending moment M at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 2 mm thick.

(a) $I = \frac{1}{12} b h^3 = \frac{1}{12}(8)(8)^3 = 341.3 \text{ mm}^4$

$c = \frac{1}{2} h = 4$ mm.

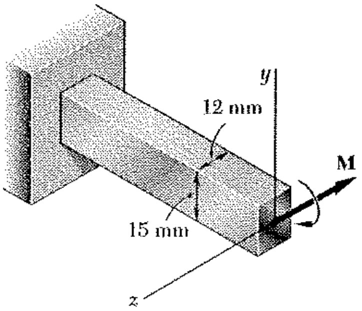
$$M_Y = \frac{\sigma_Y I}{c} = \frac{(330\times10^6)(341.3\times10^{-12})}{0.004} = 28.16 \text{Nm}$$

$M_Y = 28.16$ Nm. ◄

(b) $y_Y = c - t_p = 4 - 2 = 2$ mm

$M_p = \frac{3}{2} M_Y \left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right] = \frac{3}{2}(28.16)\left[1 - \frac{1}{3}\left(\frac{2}{4}\right)^2\right]$ $\qquad M_p = 38.72$ Nm ◄

Problem 4.71



4.71 The prismatic bar shown, made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 300$ MPa, is subjected to a couple of 162 N · m parallel to the z axis. Determine (*a*) the thickness of the elastic core, (*b*) the radius of curvature of the bar.

(a) $I = \frac{1}{12}(0.012)(0.015)^3 = 3.375 \times 10^{-9}\ m^4$

$c = \frac{1}{2}(0.015) = 0.0075\ m$

$M_Y = \frac{\sigma_Y I}{c} = \frac{(300\times10^6)(3.375\times10^{-9})}{0.0075}$

$= 135\ Nm$

$M = \frac{3}{2} M_Y \left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^3\right]$

$\frac{y_Y}{c} = \sqrt{3 - 2\frac{M}{M_Y}} = \sqrt{3 - \frac{(2)(162)}{135}} = 0.7746$

$y_Y = (0.7746)(0.0075) = 5.81\times10^{-3} m$

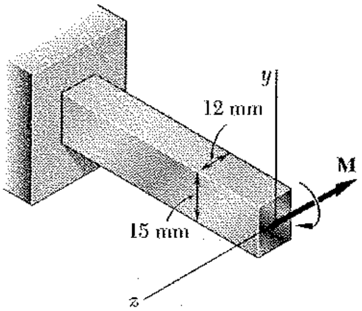
Thickness of elastic core :- $2y_Y = 11.6\ mm$ ◄

(b) $y_Y = \varepsilon_Y \rho = \frac{\sigma_Y}{E}\rho$

$\rho = \frac{y_Y E}{\sigma_Y} = \frac{(5.81\times10^{-3})(200\times10^{9})}{300\times10^6} = 3.873\ m$

$\rho = 3.9 m$ ◄

Problem 4.72



4.72 Solve Prob. 4.71, assuming that the 162 N · m couple is parallel to the *y* axis.

4.71 The prismatic bar shown, made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 300$ MPa, is subjected to a couple of 162 N · m parallel to the *z* axis. Determine (*a*) the thickness of the elastic core, (*b*) the radius of curvature of the bar.

(a) $I = \frac{1}{12}(0.015)(0.012)^3 = 2.18 \times 10^{-9}\ m^4$

$c = (\frac{1}{2})(0.012) = 0.006\ m$

$M_Y = \frac{\sigma_Y I}{c} = \frac{(300\times10^6)(2.18\times10^{-9})}{0.006}$

$= 109\ Nm$

$M = \frac{3}{2}\left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right]$

$\frac{y_Y}{c} = \sqrt{3 - 2\frac{M}{M_Y}} = \sqrt{3 - \frac{(2)(162)}{109}} = 0.1659$

$y_Y = (0.1659)(0.006) = 9.954\times10^{-4}$

Thickness of elastic core $2y_Y = 2\ mm$

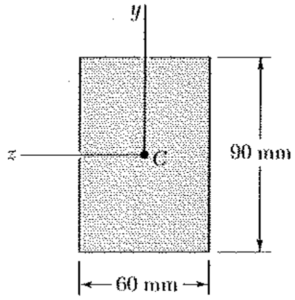
(b) $y_Y = \varepsilon_Y \rho = \frac{\sigma_Y}{E}\rho$

$\rho = \frac{y_Y E}{\sigma_Y} = \frac{(9.954\times10^{-4})(200\times10^9)}{300\times10^6} = 0.6636\ m$

$\rho = 0.67\ m$

Problem 4.73

4.73 and 4.74 A beam of the cross section shown is made of a steel which is assumed to be elastoplastic $E = 200$ GPa and $\sigma_Y = 240$ MPa. For bending about the z axis, determine the bending moment at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 30-mm thick.

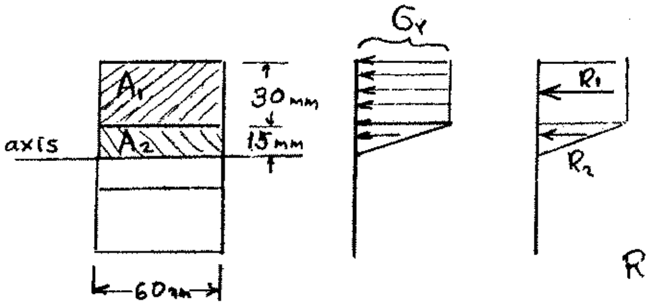


(a) $I = \frac{1}{12} b h^3 = \frac{1}{12}(60)(90)^3 = 3.645 \times 10^6 \text{ mm}^4 = 3.645 \times 10^{-6} \text{ m}^4$

$c = \frac{1}{2} h = 45 \text{ mm} = 0.045 \text{ m}$

$M_Y = \frac{\sigma_Y I}{c} = \frac{(240 \times 10^6)(3.645 \times 10^{-6})}{0.045} = 19.44 \times 10 \text{ N·m}$

$M_Y = 19.44 \text{ kN·m}$ ◄



$R_1 = \sigma_Y A_1 = (240 \times 10^6)(0.060)(0.030)$
$= 432 \times 10^3 \text{ N}$

$y_1 = 15 \text{ mm} + 15 \text{ mm} = 0.030 \text{ m}$

$R_2 = \frac{1}{2} \sigma_Y A_2 = (\frac{1}{2})(240 \times 10^6)(0.060)(0.015)$
$= 108 \times 10^3 \text{ N}$

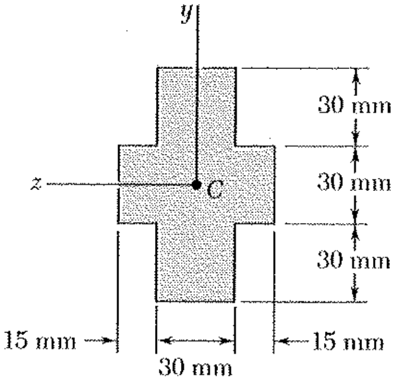
$y_2 = \frac{2}{3}(15 \text{ mm}) = 10 \text{ mm} = 0.010 \text{ m}$

(b) $M = 2(R_1 y_1 + R_2 y_2) = 2[(432 \times 10^3)(0.030) + (108 \times 10^3)(0.010)]$
$= 28.08 \times 10^3 \text{ N·m}$

$M = 28.1 \text{ kN·m}$ ◄

Problem 4.74

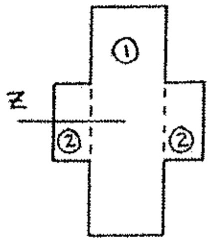
4.73 and 4.74 A beam of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 240$ MPa. For bending about the z axis, determine the bending moment at which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 30 mm thick.



(a) $I_{①} = \frac{1}{12} b_1 h_1^3$

$= \frac{1}{12}(30)(90)^3$

$= 1.8225 \times 10^6 \text{ mm}^4$



$I_{②} = \frac{1}{12} b_2 h_2^3$

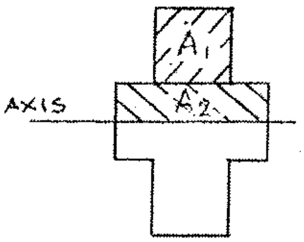
$= \frac{1}{12}(30)(30)^3 = 67.5 \times 10^3 \text{ mm}^4$

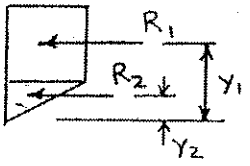
$I = I_{①} + I_{②} = 1.89 \times 10^6 \text{ mm}^4 = 1.89 \times 10^{-6} \text{ m}^4$ $\quad c = 45 \text{ mm} = 0.045 \text{ m}$

$M_Y = \frac{\sigma_Y I}{c} = \frac{(240 \times 10^6)(1.89 \times 10^{-6})}{0.045} = 10.08 \times 10^3 \text{ N·m}$

$M_Y = 10.08$ kN-m ◄

(b)





$R_1 = \sigma_Y A_1 = (240 \times 10^6)(0.030)(0.030) = 216 \times 10^3 \text{ N}$

$y_1 = 15 + 15 = 30 \text{ mm} = 0.030 \text{ m}$

$R_2 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(240 \times 10^6)(0.060)(0.015) = 108 \times 10^3 \text{ N}$

$y_2 = \frac{2}{3}(15) = 10 \text{ mm} = 0.010 \text{ m}$

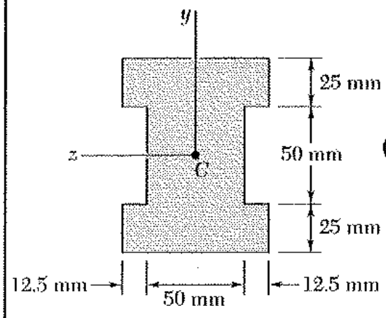
$M = 2(R_1 y_1 + R_2 y_2)$

$= 2[(216 \times 10^3)(0.030) + (108 \times 10^3)(0.015)]$

$= 16.12 \times 10^3 \text{ N-m}$ $\quad M = 16.12$ kN·m ◄

Problem 4.75

4.75 and 4.76 A beam of the cross section shown is made of a steel that is assumed to be elastoplastic $E = 200$ GPa and $\sigma_Y = 300$ MPa. For bending about the z axis, determine the bending moment and radius of curvature at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 25 mm thick.



$$\text{(a)}\quad I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(75)(25)^3 + (75)(25)(37.5)^2 = 2734375 \text{ mm}^4$$

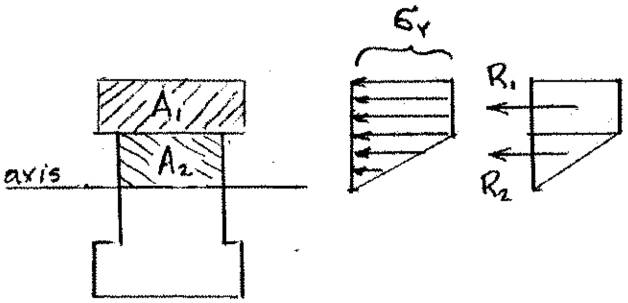
$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(50)(50)^3 = 520833 \text{ mm}^4$$

$$I_3 = I_1 = 2734375 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 5989583 \text{ mm}^4$$

$$c = 50 \text{ mm}$$

$$M_y = \frac{\sigma_Y I}{c} = \frac{(300\times10^{6})(5989583\times10^{-12})}{.050} = 35.9 \text{ kNm} \ \blacktriangleleft$$



$$R_1 = \sigma_Y A_1 = (300\times10^{6})(0.075)(0.025) = 562.5 \text{ kN}$$

$$y_1 = 25 + 12.5 = 37.5 \text{ mm.}$$

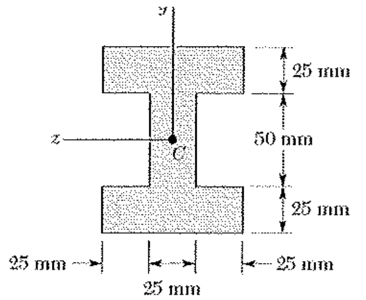
$$R_2 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(300\times10^{6})(0.05)(0.025)$$

$$= 187.5 \text{ kN}$$

$$y_2 = \frac{2}{3}(25) = 16.67 \text{ mm.}$$

$$\text{(b)}\quad M = 2(R_1 y_1 + R_2 y_2) = 2\left[(562.5)(0.0375) + (187.5)(0.01667)\right] = 48.5 \text{ kN.m} \ \blacktriangleleft$$

Problem 4.76



4.75 and 4.76 A beam of the cross section shown is made of a steel that is assumed to be elastoplastic $E = 200$ GPa and $\sigma_Y = 300$ MPa. For bending about the z axis, determine the bending moment and radius of curvature at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 25 mm thick.

(a) $I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(75)(25)^3 + (75)(25)(37.5)^2$

$= 2734375 \text{ mm}^4$

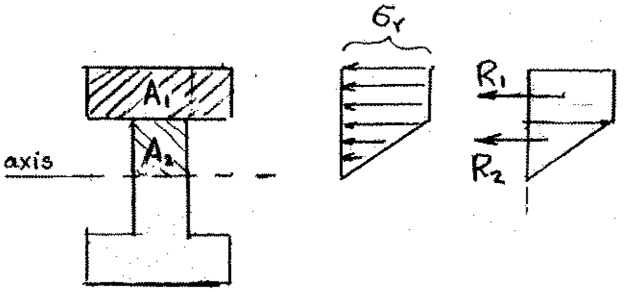
$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(25)(50)3 = 260417 \text{ mm}^4$

$I_3 = I_1 = 2734375 \text{ mm}^4$

$I = I_1 + I_2 + I_3 = 5729167 \text{ mm}^4$

$c = 50 \text{ mm}$

$M_Y = \frac{\sigma_Y I}{c} = \frac{(300 \times 10^6)(5729167 \times 10^{-12})}{0.05} = 34.4 \text{ kNm}$ ◄



$R_1 = \sigma_Y A_1 = (300 \times 10^6)(0.075)(0.025)$

$= 562.5 \text{ kN}$

$y_1 = 25 + 12.5 = 37.5 \text{ mm}$

$R_2 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(300 \times 10^6)(25 \times 10^{-3})^2$

$= 93.75 \text{ kN}$

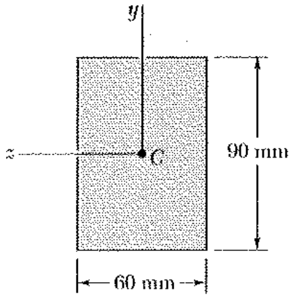
$y_2 = \frac{2}{3}(25) = 16.67 \text{ mm}$

(b) $M = 2(R_1 y_1 + R_2 y_2) = 2[(562.5)(0.0375) + (93.75)(0.01667)] = 45.3 \text{ kNm}$ ◄

Problem 4.77

4.77 through 4.80 For the beam indicated, determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

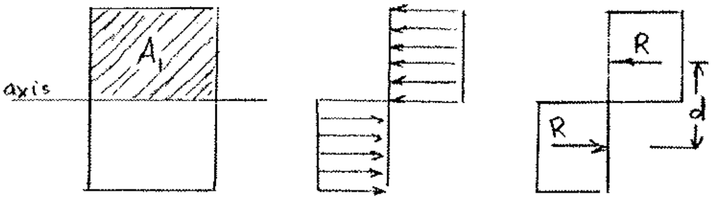
4.77 Beam of Prob. 4.73.



From PROBLEM 4.73 $E = 200$ GPa and $\sigma_Y = 240$ MPa.

$A_1 = (60)(45) = 2700 \text{ mm}^2$

$= 2700 \times 10^{-6} \text{ m}^2$



$R = \sigma_Y A_1$

$= (240 \times 10^6)(2700 \times 10^{-6})$

$= 648 \times 10^3$ N

$d = 45$ mm $= 0.045$ m

(a) $M_p = R d = (648 \times 10^3)(0.045) = 29.16 \times 10^3$ N·m $\quad M_p = 29.2$ kN·m ◄

(b) $I = \frac{1}{12} b h^3 = \frac{1}{12}(60)(90)^3 = 3.645 \times 10^6 \text{ mm}^4 = 3.645 \times 10^{-6} \text{ m}^4$

$c = 45$ mm $= 0.045$ m

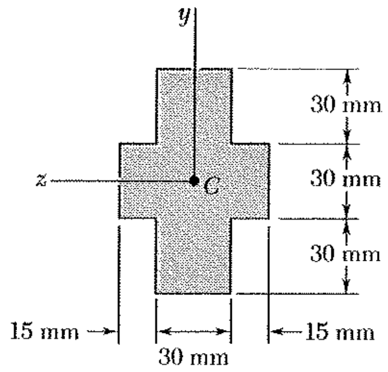
$M_Y = \frac{\sigma_Y I}{c} = \frac{(240 \times 10^6)(3.645 \times 10^{-6})}{0.045} = 19.44 \times 10^3$ N·m

$k = \frac{M_P}{M_Y} = \frac{29.16}{19.44}$ $\quad k = 1.500$ ◄

Problem 4.78

4.77 through 4.80 For the beam indicated, determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

4.78 Beam of Prob. 4.74.



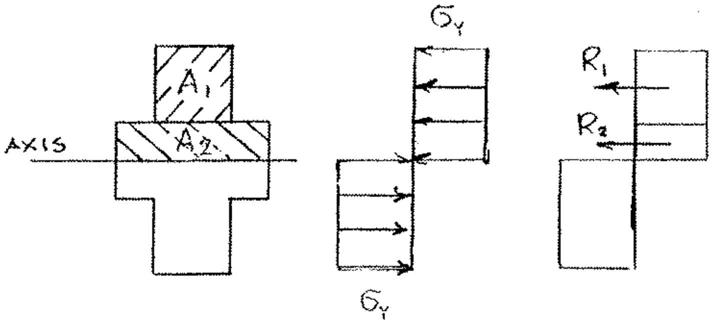
From PROBLEM 4.74 $E = 200$ GPa and $\sigma_Y = 240$ MPa

(a) $R_1 = \sigma_Y A_1$

$= (240\times10^6)(0.030)(0.030)$

$= 216\times10^3$ N

$\bar{y}_1 = 30$ mm $= 0.030$ m



$R_2 = \sigma_Y A_2$

$=(240\times10^6)(0.060)(0.015$

$= 216\times10^3$ N

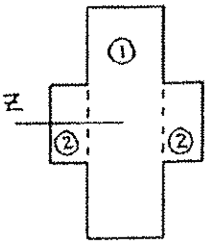
$\bar{y}_2 = 7.5$ mm $= 0.0075$ m

$M_p = 2(R_1\bar{y}_1 + R_2\bar{y}_2) = 2[(216\times10^3)(0.030) + (216\times10^3)(0.0075)]$

$= 16.20\times10^3$ N·m

$M_p = 16.20$ kN·m ◄

(b)



$I_{①} = \frac{1}{12}b_1h_1^3 = \frac{1}{12}(30)(90)^3 = 1.8225\times10^6$ mm^4

$I_{②} = \frac{1}{12}b_2h_2^3 = \frac{1}{12}(30)(30)^3 = 67.5\times10^3$ mm^4

$I = I_{①} + I_{②} = 1.89\times10^6$ mm^4 $= 1.89\times10^{-6}$ m^4

$c = 45$ mm $= 0.045$ m

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(240\times10^6)(1.89\times10^{-6})}{0.045} = 10.08\times10^3 \text{ N·m}$$

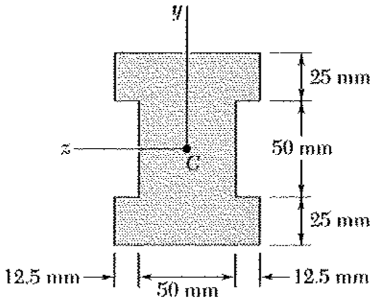
$$k = \frac{M_p}{M_y} = \frac{16.20\times10^3}{10.08\times10^3}$$

$k = 1.607$ ◄

Problem 4.79

4.77 through 4.80 For the beam indicated, determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

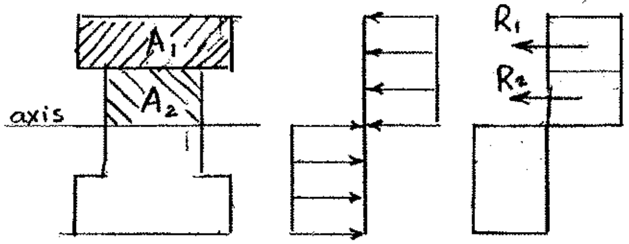
4.79 Beam of Prob. 4.75.



From PROBLEM 4.76 $E = 29\times10^6$ and $\sigma_Y = 300$ MPa.

$R_1 = \sigma_Y A_1 = (300\times10^6)(0.075)(0.025) = 562.5\ kN$

$y_1 = 25+12.5 = 37.5$ hm



$R_2 = \sigma_Y A_2 = (300\times10^6)(0.05)(0.025) = 375\ kN$

$y_2 = \frac{1}{2}(25) = 12.5$ mm

$M_P = 2(R_1y_1 + R_2y_2) = 2[(562.5)(0.0375)+(375)(0.0125)] = 51.6\ kN$ ◄

(b) $I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(75)(25)^3+(75)(25)(37.5)^2 = 2734375\ mm^4.$

$I_2 = \frac{1}{12}b_2h_2^3 = \frac{1}{12}(50)(50) = 520833\ mm^4$

$I_3 = I_1 = 2734375\ mm^4$

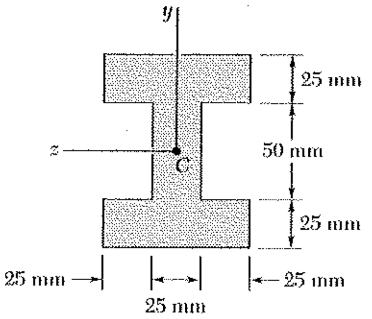
$I = I_1 + I_2 + I_3 = 5989583$

$c = 50$ mm

$M_Y = \frac{\sigma_Y I}{c} = \frac{(300\times10^6)(5989583\times10^{-12})}{0.05} = 35.9\ kNm.$

$k = \frac{M_P}{M_Y} = \frac{51.6}{35.9} = 1.437$ ◄

Problem 4.80



4.77 through 4.80 For the beam indicated, determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

4.77 Beam of Prob. 4.73.
4.78 Beam of Prob. 4.74.
4.79 Beam of Prob. 4.75.
4.80 Beam of Prob. 4.76.

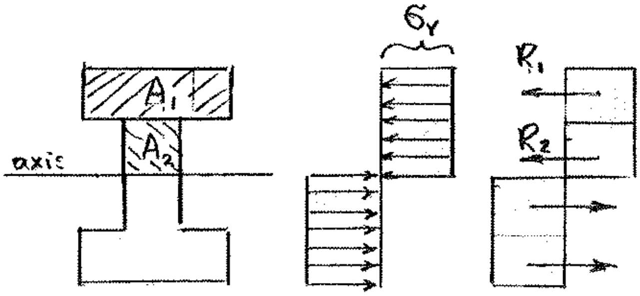
From PROBLEM 4.75 $E = 200\times10^9$ GPa and $\sigma_Y = 300$ MPa

$R_1 = \sigma_Y A_1 = (300\times10^6)(0.075)(0.025) = 562.5$ kN

$\bar{y}_1 = 25 + 12.5 = 37.5$ mm

$R_2 = \sigma_Y A_2 = (300\times10^6)(0.025)(0.025)$
$= 187.5$ kN

$\bar{y}_2 = \frac{1}{2}(25) = 12.5$ mm



$$M_p = 2(R_1 y_1 + R_2 y_2) = 2\left[(562.5)(0.0375)+(187.5)(0.0125)\right] = 46.9 \text{ kN m}$$

(b) $I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(75)(25)^3 + (75)(25)(37.5)^2 = 2734375 \text{ mm}^4.$

$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(25)(50)^3 = 260417 \text{ mm}^4$

$I_3 = I_1 = 2734375 \text{ mm}^4$

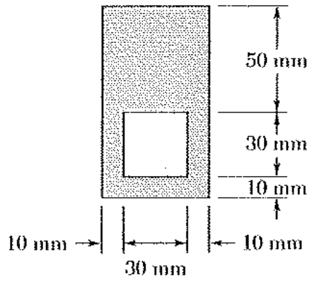
$I = I_1 + I_2 + I_3 = 5729167 \text{ mm}^4.$

$c = 50$ mm

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(300\times10^6)(5729167\times10^{-12})}{0.05} = 34.4 \text{ kNm}$$

$$k = \frac{M_P}{M_Y} = \frac{46.9}{34.4} = 1.363$$

Problem 4.81

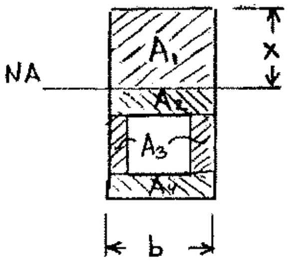


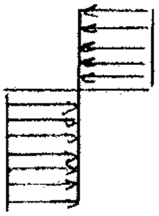
***4.81* and 4.82** Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa.

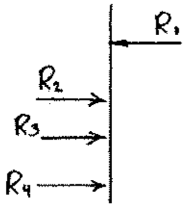
Total area: $A = (50)(90) - (30)(30) = 3600 \text{ mm}^2$

$\frac{1}{2}A = 1800 \text{ mm}^2$

$x = \frac{\frac{1}{2}A}{b} = \frac{1800}{50} = 36 \text{ mm}$







$A_1 = (50)(36) = 1800 \text{ mm}^2, \quad \bar{y}_1 = 18 \text{ mm} \qquad A_1\bar{y}_1 = 32.4 \times 10^3 \text{ mm}^3$

$A_2 = (50)(14) = 700 \text{ mm}^2, \quad \bar{y}_2 = 7 \text{ mm} \qquad A_2\bar{y}_2 = 4.9 \times 10^3 \text{ mm}^3$

$A_3 = (20)(30) = 600 \text{ mm}^2, \quad \bar{y}_3 = 29 \text{ mm} \qquad A_3\bar{y}_3 = 17.4 \times 10^3 \text{ mm}^3$

$A_4 = (50)(10) = 500 \text{ mm}^2, \quad \bar{y}_4 = 49 \text{ mm} \qquad A_4\bar{y}_4 = 24.5 \times 10^3 \text{ mm}^3$

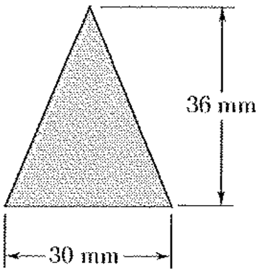
$A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3 + A_4\bar{y}_4 = 79.2 \times 10^3 \text{ mm}^3 = 79.2 \times 10^{-6} \text{ m}^3$

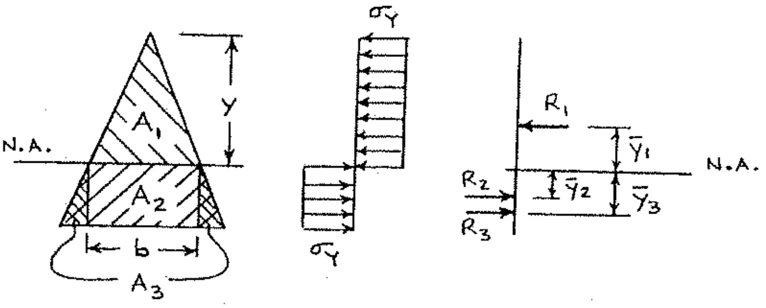
$M_p = \sigma_Y \sum A_i\bar{y}_i = (240 \times 10^6)(79.2 \times 10^{-6}) = 19.008 \times 10^3 \text{ N}\cdot\text{m}$

$M_p = 19.01 \text{ kN}\cdot\text{m}$ ◀

Problem 4.82

4.81 and 4.82 Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa.





Total area: $A = \frac{1}{2}(30)(36) = 540 \text{ mm}^2$

$\frac{1}{2}A = 270 \text{ mm}^2 = A_1$

By similar triangles, $\frac{b}{y} = \frac{30}{36}$ $b = \frac{5}{6}y$

Since $A_1 = \frac{1}{2}by = \frac{5}{12}y^2$, $y^2 = \frac{12}{5}A_1$

$y = \sqrt{\frac{12}{5}(270)} = 25.4558 \text{ mm}$ $b = 21.2132 \text{ mm}$

$A_1 = \frac{1}{2}(21.2132)(25.4558) = 270 \text{ mm}^2 = 270 \times 10^{-6} \text{ m}^2$

$A_2 = (21.2132)(36 - 25.4558) = 223.676 \text{ mm}^2 = 223.676 \times 10^{-6} \text{ m}^2$

$A_3 = A - A_1 - A_2 = 46.324 = 46.324 \times 10^{-6} \text{ m}^2$

$R_i = \sigma_Y A_i = 240 \times 10^6 A_i$

$R_1 = 64.8 \times 10^3 \text{ N}$, $R_2 = 53.6822 \times 10^3 \text{ N}$, $R_3 = 11.1178 \times 10^3 \text{ N}$

$\bar{y}_1 = \frac{1}{3}y = 8.4853 \text{ mm} = 8.4853 \times 10^{-3} \text{ m}$

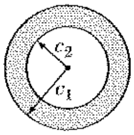
$\bar{y}_2 = \frac{1}{2}(36 - 25.4558) = 5.2721 \text{ mm} = 5.2721 \times 10^{-3} \text{ m}$

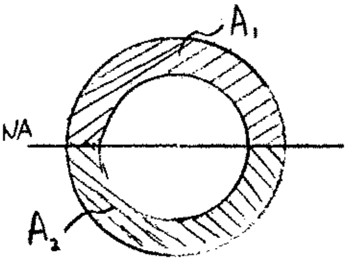
$\bar{y}_3 = \frac{2}{3}(36 - 25.4558) = 7.0295 \text{ mm} = 7.0295 \times 10^{-3} \text{ m}$

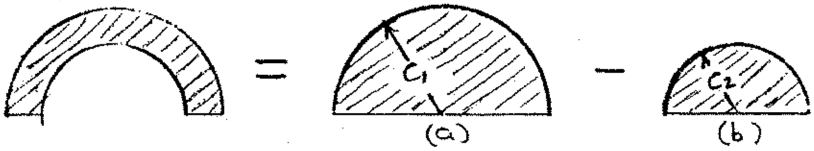
$M_p = R_1\bar{y}_1 + R_2\bar{y}_2 + R_3\bar{y}_3 = 911 \text{ N} \cdot \text{m}$ $M_p = 911 \text{ N} \cdot \text{m}$ ◄

Problem 4.83

4.83 A thick-walled pipe of the cross section shown is made of a steel that is assumed to be elastoplastic with a yield strength σ_Y. Derive an expression for the plastic moment M_p of the pipe in terms of c_1, c_2, and σ_Y.







$$A_1\bar{y}_1 = A_a\bar{y}_a - A_b\bar{y}_b$$

$$= \left(\frac{\pi}{2}c_1^2\right)\left(\frac{4c_1}{3\pi}\right) - \left(\frac{\pi}{2}c_2^2\right)\left(\frac{4c_2}{3\pi}\right)$$

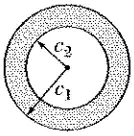
$$= \frac{2}{3}(c_1^3 - c_2)^3$$

$$A_2\bar{y}_2 = A_1\bar{y}_1 = \frac{2}{3}(c_1^3 - c_2^2)$$

$$M_p = \sigma_Y(A_1\bar{y}_1 + A_2\bar{y}_2) = \frac{4}{3}\sigma_Y(c_1^3 - c_2^3)$$ ◄

Problem 4.84

4.84 Determine the plastic moment M_p of a thick-walled pipe of the cross section shown, knowing that $c_1 = 60$ mm, $c_2 = 40$ mm, and $\sigma_Y = 240$ MPa.



See the solution to PROBLEM 4.83 for derivation of the following expression for M_p:

$$M_p = \frac{4}{3}\sigma_Y(c_1^3 - c_2^3)$$

Data: $\sigma_Y = 240$ MPa $= 240 \times 10^6$ Pa

$c_1 = 60$ mm $= 0.060$ m

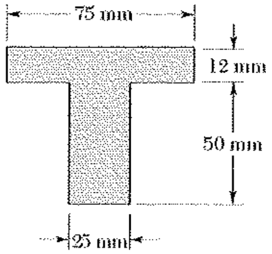
$c_2 = 40$ mm $= 0.040$ m

$$M_p = \frac{4}{3}(240\times10^6)(0.060^3 - 0.040^3) = 48.67\times10^3 \text{ N·m}$$

$$M_p = 48.6 \text{ kN·m}$$ ◄

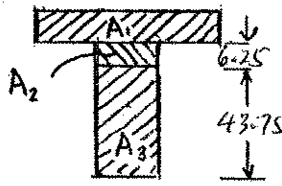
Problem 4.85

4.85 Determine the plastic moment M_p of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 330 MPa.



Total area: $A = (75)(12) + (50)(25) = 2150\ mm^2$

$\frac{1}{2}A = 1075\ mm^2$ $\qquad y_n = \frac{\frac{1}{2}A}{b} = \frac{1075}{25} = 43\ mm$



$A_1 = (75)(12) = 900\ mm^2,\ \bar{y}_1 = 12.25\ mm,\ A_1\bar{y}_1 = 11025\ mm^3$

$A_2 = (6.25)(25) = 156.25\ mm^2,\ \bar{y}_2 = 3.125\ mm\ \ A_2\bar{y}_2 = 488.3\ mm^3$

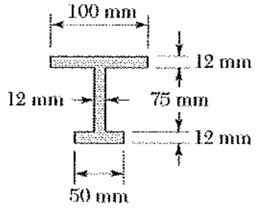
$A_3 = (43.75)(25) = 1093.75\ mm^2\ \ \bar{y}_3 = 21.875\ mm\ \ A_3\bar{y}_3 = 23925.8$

$M_p = \sigma_Y(A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3)$

$= 330\times10^6(11025 + 488.3 + 23925.8)(1\times10^{-9}) = 11.7\ kN\,m.$ ◄

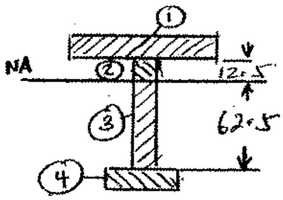
Problem 4.86

4.86 Determine the plastic moment M_p of the cross section shown assuming the steel to be elastoplastic with a yield strength of 250 MPa.



Total area: $A = (100\times12) + (75)(12) + (50)(12) = 2700\ mm^2$

$\frac{1}{2}A = 1350\ mm^2$



$A_1 = 1200\ mm^2,\quad \bar{y}_1 = 18.5,\quad A_1y_1 = 22200\ mm^3$

$A_2 = 150\ mm^2,\quad \bar{y}_2 = 6.25,\quad A_2\bar{y}_2 = 937.5\ mm^3$

$A_3 = 750\ mm^2,\quad \bar{y}_3 = 31.25,\quad A_3\bar{y}_3 = 23437.5\ mm^3$

$A_4 = 600\ mm^2,\quad \bar{y}_4 = 68.5,\quad A_4\bar{y}_4 = 41100\ mm^3$

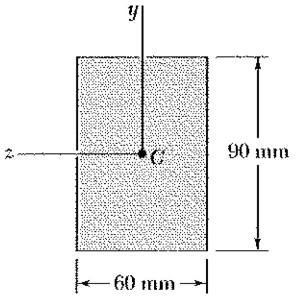
$M_p = \sigma_Y(A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3 + A_4\bar{y}_4)$

$= (250\times10^6)(22200 + 937.5 + 23437.5 + 41100)(1\times10^{-9}) = 21.9\ kN\,m.$ ◄

Problem 4.87

4.87 and 4.88 For the beam indicated, a couple of moment equal to the full plastic moment M_p is applied and then removed. Using a yield strength of 240 MPa, determine the residual stress at $y = 45$ mm.

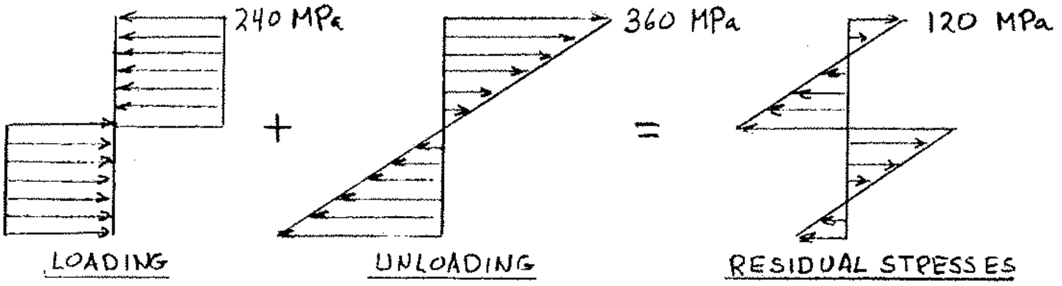
4.87 Beam of Prob. 4.73.



$M_p = 29.16 \times 10^3$ N·m (See solutions to Problems 4.73 and 4.77)

$I = 3.645 \times 10^{-6}\ \text{m}^4$, $c = 0.045$ m

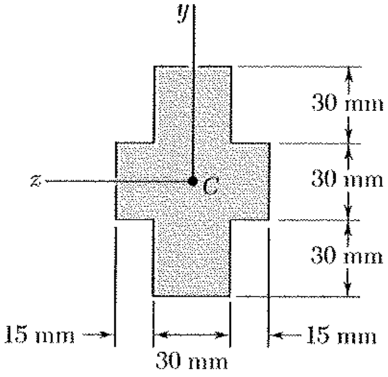
$\sigma' = \frac{M_{max} y}{I} = \frac{M_p c}{I}$ at $y = c = 45$ mm.



$$\sigma' = \frac{(29.16\times10^3)(0.045)}{3.645\times10^{-6}} = 360\times10^6\ \text{Pa}$$

$\sigma_{res} = \sigma' - \sigma_Y = 360\times10^6 - 240\times10^6 = 120\times10^6$ Pa $\qquad \sigma_{res} = 120.0$ MPa ◄

Problem 4.88



4.87 and 4.88 For the beam indicated, a couple of moment equal to the full plastic moment M_p is applied and then removed. Using a yield strength of 240 MPa, determine the residual stress at $y = 45$ mm.

4.88 Beam of Prob. 4.74.

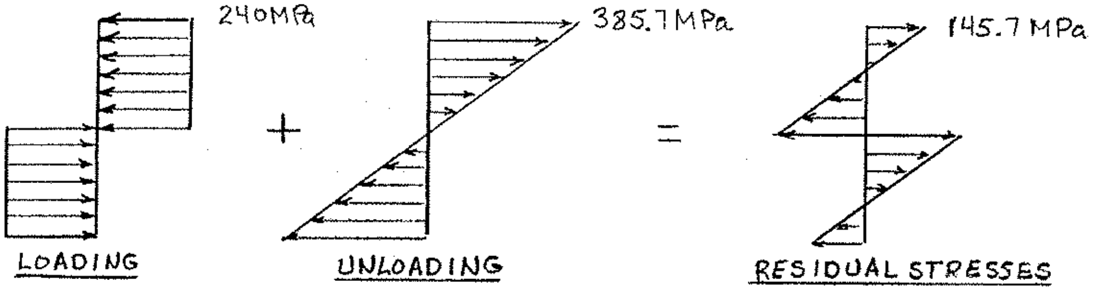
$M_p = 16.20 \times 10^3$ N·m from the solution to PROBLEM 4.78

$I = 1.86 \times 10^{-6}\ \text{m}^4$

$c = 0.045$ m

$$\sigma' = \frac{M_{max} c}{I} = \frac{M_p c}{I} = \frac{(16.20 \times 10^3)(0.045)}{1.89 \times 10^{-6}}$$

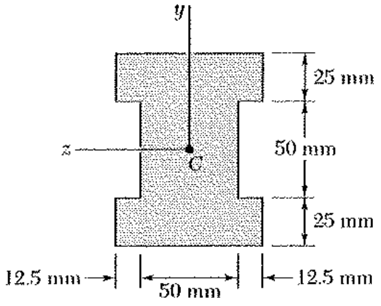
$$= 385.7 \times 10^6 \text{ Pa} = 385.7 \text{ MPa}$$



At $y = 45$ $\sigma = 240$ MPa $\sigma' = 385.7$ MPa

Residual stress $\sigma_{res} = \sigma' - \sigma_Y$ $\sigma_{res} = 145.7$ MPa ◄

Problem 4.89



4.89 and 4.90 For the beam indicated, a couple of moment equal to the full plastic moment M_p is applied and then removed. Using a yield strength of 290 MPa, determine the residual stress at (*a*) $y = 25$ mm, (*b*) $y = 50$ mm.

4.89 Beam of Prob. 4.75.

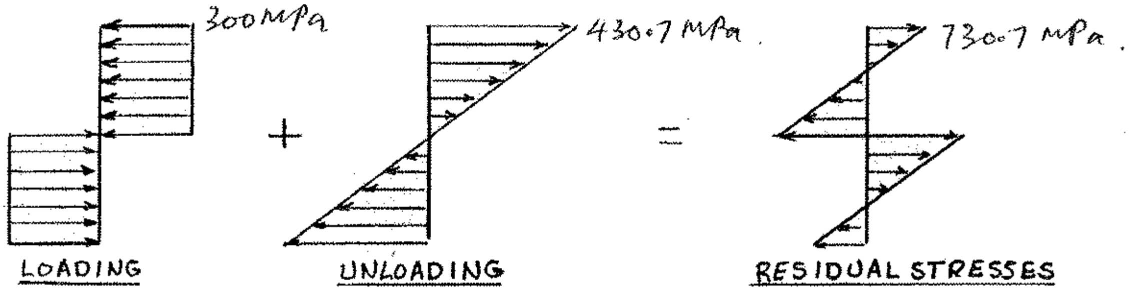
4.90 Beam of Prob. 4.76.

$M_p = 51.6$ kN (See solutions to Problems 4.76 and 4.80)

$I = 5989583\ mm^4$, $c = 50$ mm.

$$\sigma' = \frac{M_{max} Y}{I} = \frac{M_p c}{I} \quad \text{for } y = c$$

$$\sigma' = \frac{(51.6\times10^3)(0.05)}{5989583\times10^{-12}} = 430.7 \text{ MPa.}$$



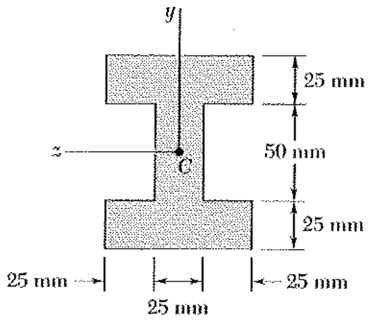
(a) At $y = 25$ mm $= \frac{1}{2}c$ $\sigma' = \frac{1}{2}(430.7) = 215.35$ MPa.

$\sigma_{res} = -300 + 215.35 = -84.65$ MPa.

(b) At $y = 50$ mm $= c$ $\sigma' = 430.7$ MPa

$\sigma_{res} = -300 + 430.7 = 730.7$ MPa.

Problem 4.90



4.89 and 4.90 For the beam indicated, a couple of moment equal to the full plastic moment M_p is applied and then removed. Using a yield strength of 290 MPa, determine the residual stress at (*a*) $y = 25$ mm, (*b*) $y = 50$ mm.

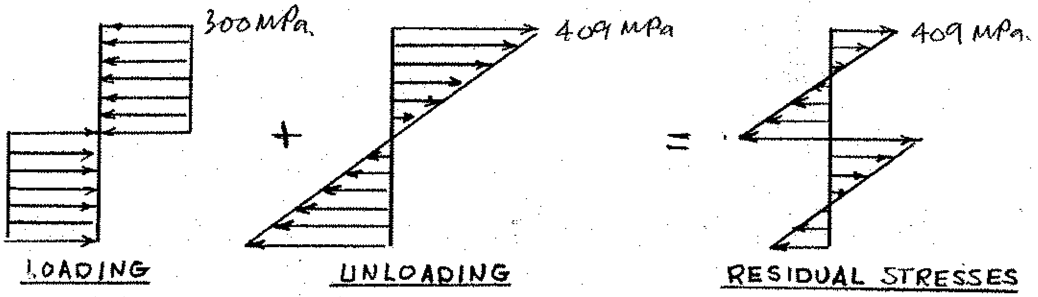
4.89 Beam of Prob. 4.75.
4.90 Beam of Prob. 4.76.

$M_p = 46.9$ kNm (See solutions to Problems 4.75 and 4.79)

$I = 5729167 \text{ mm}^4 \quad c = 50 \text{ mm}.$

$$\sigma' = \frac{M_{max} y}{I} = \frac{M_p c}{I} \quad \text{at } y = c.$$

$$\sigma' = \frac{(46.9 \times 10^3)(0.05)}{5729167 \times 10^{-12}} = 409 \text{ MPa}.$$



(a) At $y = 25 \text{ mm} = \frac{1}{2}c \qquad \sigma' = \frac{1}{2}(409) = 204.5 \text{ MPa}.$

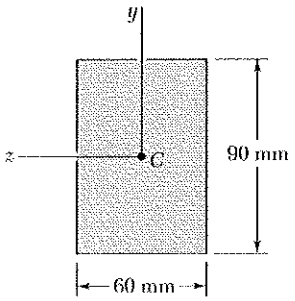
$\sigma_{res} = -300 + 204.5 = -95.5 \text{ MPa}.$

(b) At $y = 50 \text{ mm} = c \qquad \sigma' = 409 \text{ MPa}.$

$\sigma_{res} = -300 + 409 = 109 \text{ MPa}.$

Problem 4.91

4.91 A bending couple is applied to the beam of Prob. 4.73, causing plastic zones 30 mm thick to develop at the top and bottom of the beam. After the couple has been removed, determine (*a*) the residual stress at $y = 45$ mm, (*b*) the points where the residual stress is zero, (*c*) the radius of curvature corresponding to the permanent deformation of the beam.



See SOLUTION to PROBLEM 4.73 for bending couple and stress distribution during loading.

$M = 28.08 \times 10^3$ N-m $\quad y_Y = 15$ mm $= 0.015$ m

$E = 200$ GPa $\quad \sigma_Y = 240$ MPa

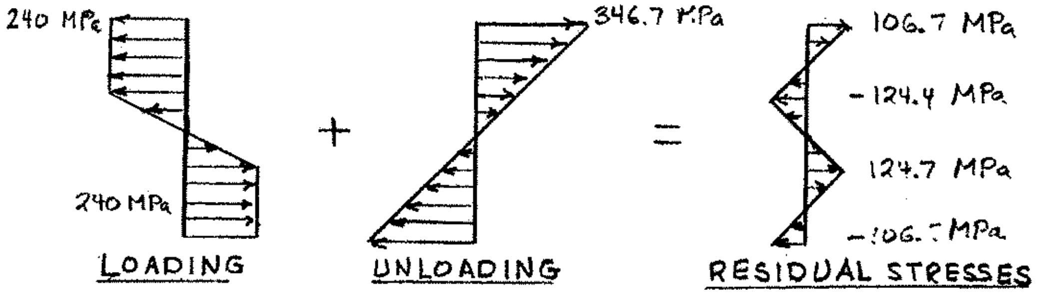
$I = 3.645 \times 10^{-6}$ m^4 $\quad c = 0.045$ m

(a) $\sigma' = \dfrac{Mc}{I} = \dfrac{(28.08 \times 10^3)(0.045)}{3.645 \times 10^{-6}} = 346.7 \times 10^6$ Pa $= 346.7$ MPa

$\sigma'' = \dfrac{My_Y}{I} = \dfrac{(28.08 \times 10^3)(0.015)}{3.645 \times 10^{-6}} = 115.6 \times 10^6$ Pa $= 115.6$ MPa

At $y = c$, $\quad \sigma_{res} = \sigma' - \sigma_Y = 346.7 - 240 \quad \sigma_{res} = 106.7$ MPa ◀

At $y = y_Y$, $\quad \sigma_{res} = \sigma'' - \sigma_Y = 115.6 - 240 \quad \sigma_{res}$ -124.4 MPa



(b) $\sigma_{res} = 0 \quad \therefore \quad \dfrac{My_0}{I} - \sigma_Y = 0$

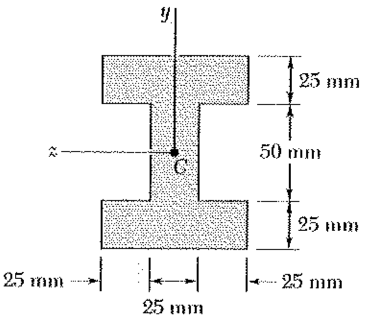
$y_0 = \dfrac{I\sigma_Y}{M} = \dfrac{(3.645 \times 10^{-6})(240 \times 10^6)}{28.08 \times 10^3} = 31.15 \times 10^{-3}$ m $= 31.15$ mm

ans. $y_0 = -31.15$ mm, 0, 31.15 mm ◀

(c) At $y = y_Y$, $\quad \sigma_{res} = -124.4 \times 10^6$ Pa

$\sigma = -\dfrac{Ey}{\rho} \quad \therefore \quad \rho = -\dfrac{Ey}{\sigma} = \dfrac{(200 \times 10^9)(0.015)}{-124.4 \times 10^6} \quad \rho = 24.1$ m ◀

Problem 4.92



4.92 A bending couple is applied to the beam of Prob. 4.76, causing plastic zones 50 mm thick to develop at the top and bottom of the beam. After the couple has been removed, determine (*a*) the residual stress at $y = 50$ mm, (*b*) the points where the residual stress is zero, (*c*) the radius of curvature corresponding to the permanent deformation of the beam.

See SOLUTION to PROBLEM 4.75 for bending couple and stress distribution

$M = 45.3$ kNm. $\qquad y_Y = 25$ mm.

$E = 200$ GPa $\qquad \sigma_Y = 300$ MPa.

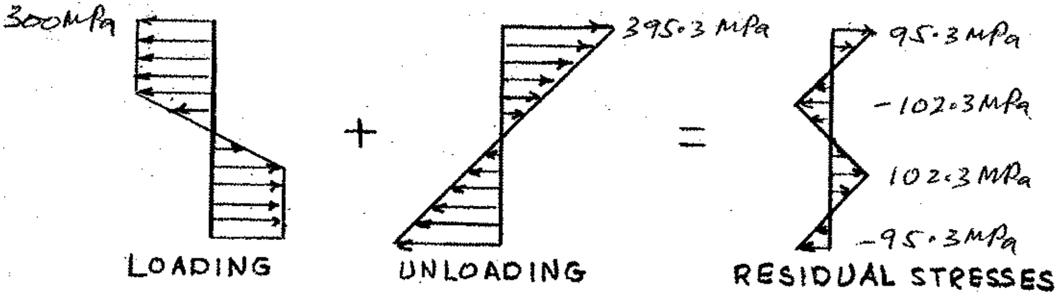
$I = 5729167$ mm^4 $\qquad c = 50$ mm.

(a) $\sigma' = \dfrac{Mc}{I} = \dfrac{(45.3\times10^3)(0.05)}{5729167\times10^{-12}} = 395.3$ MPa.

$\sigma'' = \dfrac{My_Y}{I} = \dfrac{(45.3\times10^3)(0.025)}{5729167\times10^{-12}} = 197.7$ MPa.

At $y = c$ $\qquad \sigma_{res} = \sigma' - \sigma_Y = 395.3 - 300 = 95.3$ MPa

At $y = y_Y$ $\qquad \sigma_{res} = \sigma'' - \sigma_Y = 197.7 - 300 = -102.3$ MPa



(b) $\sigma_{res} = 0 \quad \therefore \quad \dfrac{My_o}{I} - \sigma_Y = 0$

$y_o = \dfrac{I\sigma_Y}{M} = \dfrac{(5729167\times10^{-12})(300\times10^6)}{45300} = 0.0379$ m.

ans. $\quad y_o = -37.9$ mm, 0, 37.9 mm.

(c) At $y = y_Y$, $\quad \sigma_{res} = -102.3$ MPa

$\sigma = -\dfrac{Ey}{\rho} \quad \therefore \quad \rho = -\dfrac{Ey}{\sigma} = \dfrac{(200\times10^9)(0.025)}{102.3\times10^6} = 48.9$ m

Problem 4.93

***4.93** A rectangular bar that is straight and unstressed is bent into an arc of circle of radius ρ by two couples of moment M. After the couples are removed, it is observed that the radius of curvature of the bar is ρ_R. Denoting by ρ_Y the radius of curvature of the bar at the onset of yield, show that the radii of curvature satisfy the following relation:

$$\frac{1}{\rho_R}=\frac{1}{\rho}\left\{1-\frac{3}{2}\frac{\rho}{\rho_Y}\left[1-\frac{1}{3}\left(\frac{\rho}{\rho_Y}\right)^2\right]\right\}$$

$$\frac{1}{\rho_Y}=\frac{M_Y}{EI}, \qquad M=\frac{3}{2}M_Y\left(1-\frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right) \qquad \text{Let } m \text{ denote } \frac{M}{M_Y}$$

$$m=\frac{M}{M_Y}=\frac{3}{2}\left(1-\frac{\rho^2}{\rho_Y^2}\right) \qquad \therefore \qquad \frac{\rho^2}{\rho_Y^2}=3-2m$$

$$\frac{1}{\rho_R}=\frac{1}{\rho}-\frac{M}{EI}=\frac{1}{\rho}-\frac{mM_Y}{EI}=\frac{1}{\rho}-\frac{m}{\rho_Y}$$

$$=\frac{1}{\rho}\left\{1-\frac{\rho}{\rho_Y}m\right\}=\frac{1}{\rho}\left\{1-\frac{3}{2}\frac{\rho}{\rho_Y}\left(1-\frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right)\right\} \qquad ◄$$

Problem 4.94

4.94 A solid bar of rectangular cross section is made of a material that is assumed to be elastoplastic. Denoting by M_Y and ρ_Y, respectively, the bending moment and radius of curvature at the onset of yield, determine (*a*) the radius of curvature when a couple of moment $M = 1.25\ M_Y$ is applied to the bar, (*b*) the radius of curvature after the couple is removed. Check the results obtained by using the relation derived in Prob. 4.93.

(a) $$\frac{1}{\rho_Y}=\frac{M_Y}{EI}, \qquad M=\frac{3}{2}M_Y\left(1-\frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right) \qquad \text{Let } m=\frac{M}{M_Y}=1.25$$

$$m=\frac{M}{M_Y}=\frac{3}{2}\left(1-\frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right) \qquad \frac{\rho}{\rho_Y}=\sqrt{3-2m}=0.70711$$

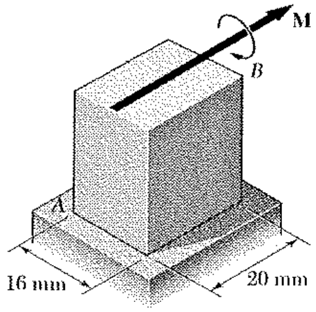
$$\rho=0.70711\ \rho_Y \qquad ◄$$

(b) $$\frac{1}{\rho_R}=\frac{1}{\rho}-\frac{M}{EI}=\frac{1}{\rho}-\frac{mM_Y}{EI}=\frac{1}{\rho}-\frac{m}{\rho_Y}=\frac{1}{0.70711\rho_Y}-\frac{1.25}{\rho_Y}$$

$$=\frac{0.16421}{\rho_Y} \qquad \rho_R=6.09\ \rho_Y \qquad ◄$$

Problem 4.95

4.95 The prismatic bar AB is made of a steel that is assumed to be elastoplastic and for which $E = 200$ GPa. Knowing that the radius of curvature of the bar is 2.4 m when a couple of moment $M = 350$ N · m is applied as shown, determine (*a*) the yield strength of the steel, (*b*) the thickness of the elastic core of the bar.



$$M = \frac{3}{2} M_Y \left(1 - \frac{1}{3} \frac{\rho^2}{\rho_Y^2} \right)$$

$$= \frac{3}{2} \frac{\sigma_Y I}{c} \left(1 - \frac{1}{3} \frac{\rho^2 \sigma_Y^2}{E^2 c^2} \right)$$

$$= \frac{3}{2} \frac{\sigma_Y b (2c)^3}{12 c} \left(1 - \frac{1}{3} \frac{\rho^2 \sigma_Y^2}{E^2 c^2} \right)$$

$$= \sigma_Y b c^2 \left(1 - \frac{1}{3} \frac{\rho^2 \sigma_Y^2}{E^2 c^2} \right)$$

(a) $bc^2 \sigma_y \left(1 - \frac{\rho^2 \sigma_Y^2}{3 E^2 c^2} \right) = M$ Cubic equation for σ_Y

Data: $E = 200 \times 10^9$ Pa, $M = 420$ N·m $\rho = 2.4$ m

$b = 20$ mm $= 0.020$ m $c = \frac{1}{2} h = 8$ mm $= 0.008$ m

$$(1.28 \times 10^{-6}) \sigma_Y \left[1 - 750 \times 10^{-21} \sigma_Y^2 \right] = 350$$

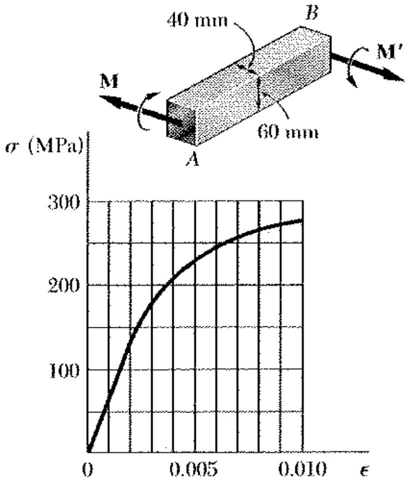
$$\sigma_Y \left[1 - 750 \times 10^{-21} \sigma_Y^2 \right] = 273.44 \times 10^6$$

Solving by trial $\sigma_Y = 292 \times 10^6$ Pa $\sigma_Y = 292$ MPa ◀

(b) $y_Y = \frac{\sigma_Y \rho}{E} = \frac{(292 \times 10^6)(2.4)}{200 \times 10^9} = 3.504 \times 10^{-3}$ m $= 3.504$ mm

thickness of elastic core $= 2 y_Y = 7.01$ mm ◀

Problem 4.96



4.96 The prismatic bar *AB* is made of an aluminum alloy for which the tensile stress-strain diagram is as shown. Assuming that the σ-ε diagram is the same in compression as in tension, determine (*a*) the radius of curvature of the bar when the maximum stress is 250 MPa, (*b*) the corresponding value of the bending moment. (*Hint*: For part *b*, plot σ versus *y* and use an approximate method of integration.)

(a) $\sigma_m = 250 \text{ MPa} = 250\times10^{6} \text{ Pa}$

$\epsilon_m = 0.0064$ from curve

$c = \frac{1}{2}h = 30 \text{ mm} = 0.030 \text{ m}$

$b = 40 \text{ mm} = 0.040 \text{ m}$

$$\frac{1}{\rho} = \frac{\epsilon_m}{c} = \frac{0.0064}{0.030} = 0.21333 \text{ m}^{-1}$$

$\rho = 4.69 \text{ m}$ ◄

(b) Strain distribution. $\epsilon = -\epsilon_m \frac{y}{c} = -\epsilon_m u$ where $u = \frac{y}{c}$

Bending couple.

$$M = -\int_{-c}^{c} y\sigma b\,dy = 2b\int_0^c y|\sigma|\,dy = 2bc^2\int_0^1 u|\sigma|\,du = 2bc^2 J$$

where the integral J is given by $\int_0^1 u|\sigma|\,du$

Evaluate J using a method of numerial integration. If Simpson's rule is used, the integration formula is

$$J = \frac{\Delta u}{3}\sum w u|\sigma|$$

where w is a weighting factor. Using $\Delta u = 0.25$ we get the values given in the table below:

u	$\lvert\epsilon\rvert$	$\lvert\sigma\rvert$, (MPa)	$u\lvert\sigma\rvert$, (MPa)	w	$wu\lvert\sigma\rvert$, (MPa)
0	0	0	0	1	0
0.25	0.0016	110	27.5	4	110
0.5	0.0032	180	90	2	180
0.75	0.0048	225	168.75	4	675
1.00	0.0064	250	250	1	250
					1215

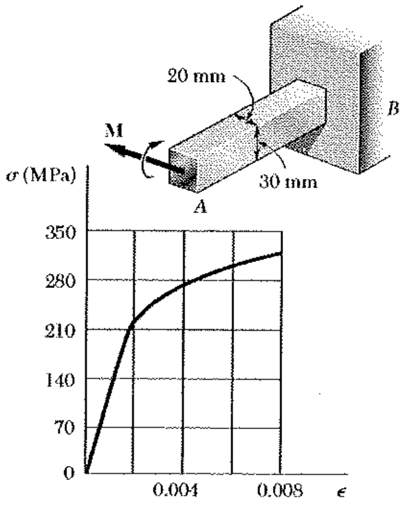
$\leftarrow \sum wu|\sigma|$

$$J = \frac{(0.25)(1215)}{3} = 101.25 \text{ MPa} = 101.25\times10^{6} \text{ Pa}$$

$M = (2)(0.040)(0.030)^2(101.25\times10^{6}) = 7.29\times10^{3} \text{ N}\cdot\text{m}$ $M = 7.29 \text{ kN}\cdot\text{m}$ ◄

Problem 4.97

4.97 The prismatic bar AB is made of a bronze alloy for which the tensile stress-strain diagram is as shown. Assuming that the σ-ϵ diagram is the same in compression as in tension, determine (*a*) the maximum stress in the bar when the radius of curvature of the bar is 2.5 m, (*b*) the corresponding value of the bending moment. (See hint given in Prob. 4.96.)



(a) $\rho = 2500$ mm, $b = 20$ mm, $c = 15$ mm.

$$\varepsilon_m = \frac{c}{\rho} = \frac{15}{2500} = 0.006$$

From the curve $\sigma_m = 300$ MPa. ◄

(b) Strain distribution $\varepsilon = -\varepsilon_m \frac{y}{c} = -\varepsilon_m u$ where $u = \frac{y}{c}$

Bending couple

$$M = -\int_{-c}^{c} y\sigma b\,dy = 2b\int_0^c y|\sigma|dy = 2bc^2\int_0^1 u|\sigma|\,du = 2bc^2 J$$

where the integral J is given by $\int_0^1 u\,|\sigma|\,du$

Evaluate J using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$J = \frac{\Delta u}{3}\sum w\,u\,|\sigma|$$

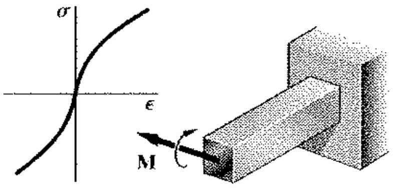
where w is a weighting factor. Using $\Delta u = 0.25$ we get the values given in the table below:

u	$\lvert\varepsilon\rvert$	$\lvert\sigma\rvert$, MPa	$u\lvert\sigma\rvert$, MPa	w	$wu\lvert\sigma\rvert$, MPa
0	0	0	0	1	0
0.25	0.0015	175	43.8	4	175
0.5	0.003	252	126	2	252
0.75	0.0045	280	210	4	840
1.00	0.006	300	300	1	300
					1567 ← $\Sigma\, w u \lvert\sigma\rvert$

$$J = \frac{(0.25)(1567)}{3} = 130.6 \text{ MPa}$$

$$M = (2)(0.02)(0.015)^2(130.6 \times 10^6) = 1.175 \text{ kN/m}.$$ ◄

Problem 4.98



4.98 A prismatic bar of rectangular cross section is made of an alloy for which the stress-strain diagram can be represented by the relation $\varepsilon = k\sigma^n$ for $\sigma > 0$ and $\varepsilon = -|k\sigma^n|$ for $\sigma < 0$. If a couple **M** is applied to the bar, show that the maximum stress is

$$\sigma_m = \frac{1+2n}{3n}\frac{Mc}{I}$$

Strain distribution: $\varepsilon = -\varepsilon_m \frac{y}{c} = -\varepsilon_m u$ where $u = \frac{y}{c}$

Bending couple.

$$M = -\int_{-c}^{c} y\sigma\, b\,dy = 2b\int_0^c y|\sigma|\,dy = 2bc^2\int_0^1 \frac{y}{c}|\sigma|\frac{dy}{c}$$

$$= 2bc^2\int_0^1 u|\sigma|\,du$$

For $\varepsilon = K\sigma^n$, $\varepsilon_m = K\sigma_m$

$$\frac{\varepsilon}{\varepsilon_m} = u = \left(\frac{\sigma}{\sigma_m}\right)^n \qquad \therefore\ |\sigma| = \sigma_m u^{\frac{1}{n}}$$

Then, $$M = 2bc^2\int_0^1 u\sigma_m u^{\frac{1}{n}}\,du = 2bc^2\sigma_m\int_0^1 u^{1+\frac{1}{n}}\,du$$

$$= 2bc^2\sigma_m \left.\frac{u^{2+\frac{1}{n}}}{2+\frac{1}{n}}\right|_0^1 = \frac{2n}{2n+1}bc^2\sigma_m$$

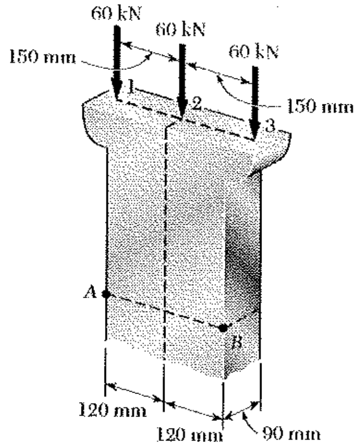
$$\sigma_m = \frac{2n+1}{2}\frac{M}{bc^2}$$

Recall $\frac{I}{c} = \frac{1}{12}\frac{b(2c)^3}{c} = \frac{2}{3}bc^2 \qquad \frac{1}{bc^2} = \frac{2}{3}\frac{c}{I}$

$$\sigma_m = \frac{2n+1}{3n}\frac{Mc}{I}$$

Problem 4.99

4.99 Determine the stress at points *A* and *B*, (*a*) for the loading shown, (*b*) if the 60-kN loads are applied at points 1 and 2 only.



(a) Loading is centric.

$P = 180 \text{ kN} = 180\times10^3 \text{ N}$

$A = (90)(240) = 21.6\times10^3 \text{ mm}^2 = 21.6\times10^{-6} \text{ m}^2$

At A and B $\quad \sigma = -\frac{P}{A} = -\frac{180\times10^3}{21.6\times10^{-3}} = -8.33\times10^6 \text{ Pa}$

$= -8.33 \text{ MPa}$ ◀

(b) Eccentric loading

$P = 120 \text{ kN} = 120\times10^3$

$M = (60\times10^3)(150\times10^{-3}) = 9.0\times10^3 \text{ N}\cdot\text{m}$

$I = \frac{1}{12}bh^3 = \frac{1}{12}(90)(240)^3 = 103.68\times10^6 \text{ mm}^4 = 103.68\times10^{-6} \text{ m}^4$

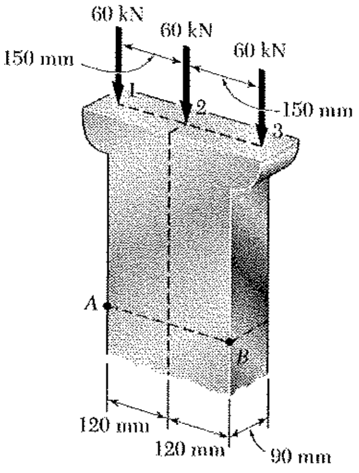
$c = 120 \text{ mm} = 0.120 \text{ m}$

At A $\quad \sigma_A = -\frac{P}{A} - \frac{Mc}{I} = -\frac{120\times10^3}{21.6\times10^{-3}} - \frac{(9.0\times10^3)(0.120)}{103.68\times10^{-6}} = -15.97\times10^6 \text{ Pa} = -15.97 \text{ MPa}$ ◀

At B $\quad \sigma_B - \frac{P}{A} + \frac{Mc}{I} = -\frac{120\times10^3}{21.6\times10^{-3}} + \frac{(9.0\times10^3)(0.120)}{103.68\times10^{-6}} = 4.86\times10^6 \text{ Pa} = 4.86 \text{ MPa}$ ◀

Problem 4.100

4.100 Determine the stress at points *A* and *B*, (*a*) for the loading shown, (*b*) if the 60-kN loads applied at points 2 and 3 are removed.



(a) Loading is centric. $\quad P = 180 \text{ kN}$

$A = (0.09)(0.24) = 0.0216 \text{ m}^2$

At A and B $\quad \sigma = -\frac{P}{A}$

$\sigma = -\frac{180\times10^3}{0.0216} \qquad \sigma = -8.3 \text{ MPa}$ ◀

(b) Eccentric loading. $\quad P = 60 \text{ kN}$

$M = (60)(0.15) = 9 \text{ kNm}$

$I = \frac{1}{12}bh^3 = \frac{1}{12}(0.09)(0.24)^3 = 103.68\times10^{-6} \text{ m}^4$

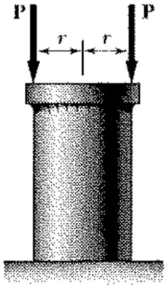
$c = 0.12 \text{ m}$

At A $\quad \sigma = -\frac{P}{A} - \frac{Mc}{I} = -\frac{60\times10^3}{0.0216} - \frac{(9\times10^3)(0.12)}{103.68\times10^{-6}} = -13.19 \text{ MPa} \qquad \sigma_A = -13.2 \text{ MPa}$ ◀

At B $\quad \sigma = -\frac{P}{A} + \frac{Mc}{I} = -\frac{60\times10^3}{0.0216} + \frac{(9\times10^3)(0.12)}{103.68\times10^{-6}} = 7.64 \text{ MPa} \qquad \sigma_B = 7.6 \text{ MPa}$ ◀

Problem 4.101

4.101 Two forces **P** can be applied separately or at the same time to a plate that is welded to a solid circular bar of radius *r*. Determine the largest compressive stress in the circular bar, (*a*) when both forces are applied, (*b*) when only one of the forces is applied.



For a solid circular section $A = \pi r^2$, $I = \frac{\pi}{4} r^4$, $c = r$

Compressive stress $\sigma = -\frac{F}{A} - \frac{Mc}{I}$

$$= -\frac{F}{\pi r^2} - \frac{4M}{\pi r^3}$$

(a) Both forces applied. $F = 2P$, $M = 0$

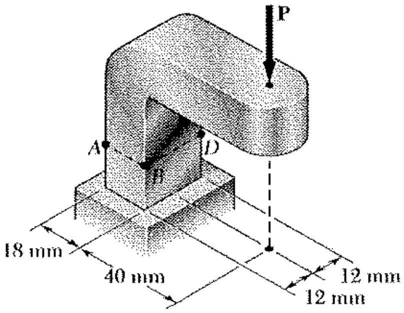
$$\sigma = -2P/\pi r^2$$

(b) One force applied. $F = P$, $M = Pr$

$$\sigma = -\frac{F}{\pi r^2} - \frac{4Pr}{\pi r^3} \qquad \sigma = -5P/\pi r^2$$

Problem 4.102

4.102 Knowing that the magnitude of the vertical force **P** is 2 kN, determine the stress at (*a*) point *A*, (*b*) point *B*.



$$A = (24)(18) = 432 \text{ mm}^2 = 432 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(24)(18)^3 = 11.664 \times 10^3 \text{ mm}^4 = 11.664 \times 10^{-9} \text{ m}^4$$

$$c = \frac{1}{2}(18) = 9 \text{ mm} = 0.009 \text{ m}$$

$$e = \frac{1}{2}(18) + 40 = 49 \text{ mm} = 0.049 \text{ m}$$

$$M = Pe = (2 \times 10^3)(0.049) = 98 \text{ N·m}$$

(a) $\sigma_A = -\frac{P}{A} + \frac{Mc}{I} = -\frac{2 \times 10^3}{432 \times 10^{-6}} + \frac{(98)(0.009)}{11.664 \times 10^{-9}} = 71.0 \times 10^6 \text{ Pa}$

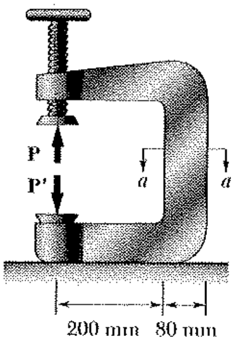
$$\sigma_A = 71.0 \text{ MPa}$$

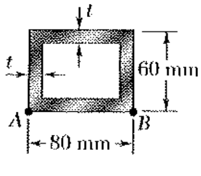
(b) $\sigma_B = -\frac{P}{A} - \frac{Mc}{I} = -\frac{2 \times 10^3}{432 \times 10^{-6}} - \frac{(98)(0.009)}{11.664 \times 10^{-9}} = -80.2 \times 10^6 \text{ Pa}$

$$\sigma_B = -80.2 \text{ MPa}$$

Problem 4.103

4.103 The vertical portion of the press shown consists of a rectangular tube of wall thickness $t = 10$ mm. Knowing that the press has been tightened on wooden planks being glued together until $P = 20$ kN, determine the stress at (*a*) point *A*, (*b*) point *B*.





Section *a-a*

Rectangular cutout is 60 mm × 40 mm.

$A = (80)(60) - (60)(40) = 2.4\times10^{3}\ mm^2 = 2.4\times10^{-3}\ m^2$

$I = \frac{1}{12}(60)(80)^3 - \frac{1}{12}(40)(60)^3 = 1.84\times10^{6}\ mm^4$

$= 1.84\times10^{-6}\ m^4$

$c = 40\ mm = 0.040\ m \qquad e = 200 + 40 = 240\ mm = 0.240\ m$

$M = Pe = (20\times10^3)(0.240) = 4.8\times10^3\ N\cdot m$

(a) $\sigma_A = \frac{P}{A} + \frac{Mc}{I} = \frac{20\times10^3}{2.4\times10^{-3}} + \frac{(4.8\times10^3)(0.040)}{1.89\times10^{-6}} = 112.7\times10^6\ Pa$

$\sigma_A = 112.8\ MPa$ ◀

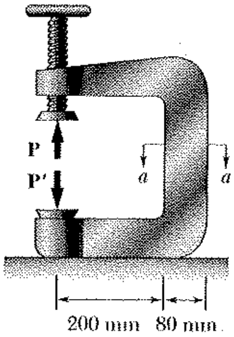
(b) $\sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{20\times10^3}{2.4\times10^{-3}} - \frac{(4.8\times10^3)(0.040.}{1.84\times10^{-6}} = 96.0\ 10^6\ Pa$

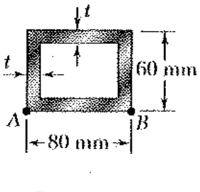
$\sigma_B = -96.0\ MPa$ ◀

Problem 4.104

4.104 Solve Prob. 4.103, assuming that $t = 8$ mm.

4.103 The vertical portion of the press shown consists of a rectangular tube of wall thickness $t = 10$ mm. Knowing that the press has been tightened on wooden planks being glued together until $P = 20$ kN, determine the stress at (*a*) point *A*, (*b*) point *B*.





Section *a-a*

Rectangular cutout is 64 mm × 44 mm

$A = (80)(60) - (64)(44) = 1.984\times10^{3}\ mm^2$

$= 1.984\times10^{-3}\ m^2$

$I = \frac{1}{12}(60)(80)^3 - \frac{1}{12}(44)(64)^3 = 1.59881\times10^{6}\ mm^2$

$= 1.59881\times10^{-6}\ m^4$

$c = 40\ mm = 0.004\ m \qquad e = 200 + 40 = 240\ mm = 0.240\ m$

$M = Pe = (20\times10^3)(0.240) = 4.8\times10^3\ N\cdot m$

(a) $\sigma_A = \frac{P}{A} + \frac{Mc}{I} = \frac{20\times10^3}{1.984\times10^{-3}} + \frac{(4.8\times10^3)(0.040)}{1.59881\times10^{-6}} = 130.2\times10^6\ Pa$

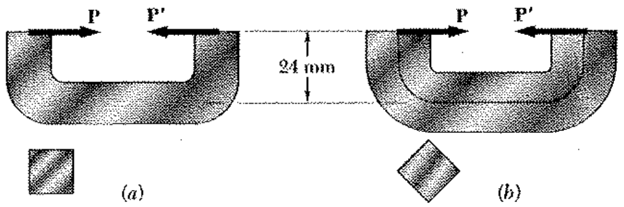
$\sigma_A = 130.2\ MPa$ ◀

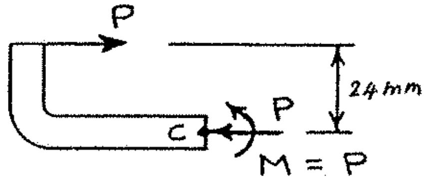
(b) $\sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{20\times10^3}{1.984\times10^{-3}} - \frac{(4.8\times10^3)(0.040)}{1.59881\times10^{-6}} = -110.0\times10^6\ Pa$

$\sigma_B = -110.0\ MPa$ ◀

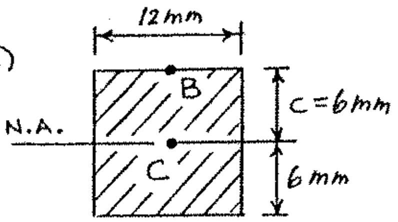
Problem 4.105

4.105 Portions of a 12 × 12 mm square bar have been bent to form the two machine components shown. Knowing that the allowable stress is 105 MPa, determine the maximum load that can be applied to each component.

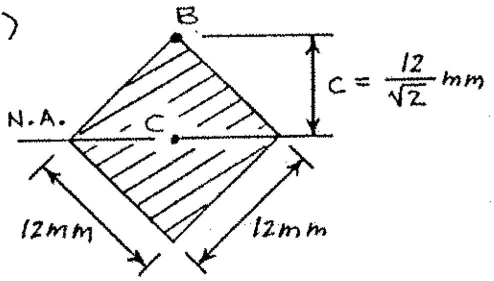




(a)



(b)



The maximum stress occurs at point B

$$\sigma_B = -105 \text{ MPa}$$

$$\sigma_B = -\frac{P}{A} - \frac{Mc}{I} = -\frac{P}{A} - \frac{Pec}{I} = -KP$$

where $K = \frac{1}{A} + \frac{ec}{I}$

$$A = (0.012)(0.012) = 144\times10^{-6} \, m^2$$

$$I = \frac{1}{12}(0.012)(0.012)^3 = 1.728\times10^{-9} \, m^4$$

$$e = 0.024 \, m$$

(a) $c = 0.006 \, m$

$$K = \frac{1}{144\times10^{-6}} + \frac{(0.024)(0.006)}{1.728\times10^{-9}} = 90277.8 \, m^{-2}$$

$$P = -\frac{\sigma_B}{K} = -\frac{(-105\times10^6)}{90277.8}$$

$$P = 1163 \, N$$

(b) $c = \frac{0.012}{\sqrt{2}} = 0.00848 \, m$

For a square I is the same for all centroidal axes.

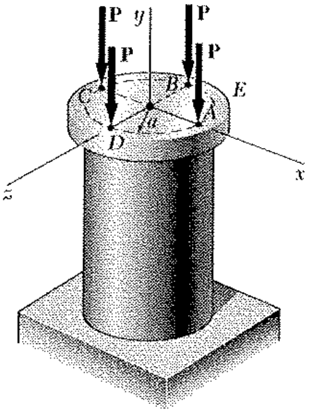
$$I = 1.728\times10^{-9} \, m^4$$

$$K = \frac{1}{144\times10^{-6}} + \frac{(0.024)(0.00848)}{1.728\times10^{-9}} = 124722.2 \, m^{-2}$$

$$P = -\frac{\sigma_B}{K} = -\frac{(-105\times10^6)}{124722.2}$$

$$P = 842 \, N$$

Problem 4.106



4.106 The four forces shown are applied to a rigid plate supported by a solid steel post of radius *a*. Knowing that *P* = 100 kN and *a* = 40 mm, determine the maximum stress in the post when (*a*) the force at *D* is removed, (*b*) the forces at *C* and *D* are removed.

For a solid circular section of radius a

$A = \pi a^2 \qquad I = \frac{\pi}{4} a^4$

(a) Centric force. $F = 4P, \quad M_x = M_z = 0$

$\sigma = -\frac{F}{A} = -\frac{4P}{\pi a^2}$

(b) Force at D is removed.

$F = 3P \qquad M_x = -Pa, \qquad M_z = 0$

$\sigma = -\frac{F}{A} - \frac{M_x z}{I} = -\frac{3P}{\pi a^2} - \frac{(-Pa)(-a)}{\frac{\pi}{4} a^2} = -\frac{7P}{\pi a^2}$

(c) Forces at C and D are removed.

$F = 2P \qquad M_x = -Pa \qquad M_z = -Pa$

Resultant bending couple $M = \sqrt{M_x^2 + M_z^2} = \sqrt{2}\, Pa$

$\sigma = -\frac{F}{A} - \frac{Mc}{I} = -\frac{2P}{\pi a^2} - \frac{\sqrt{2} Pa\, a}{\frac{\pi}{4} a^2} = -\frac{2 + 4\sqrt{2}}{\pi} \frac{P}{a^2} = -2.437\, P/a^2$

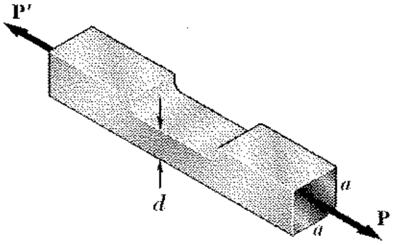
Numerical data: $P = 100 \times 10^3$ N $\qquad a = 0.040$ m

Answers: (a) $\sigma = -\frac{(4)(100 \times 10^3)}{\pi (0.040)^2} = -79.6 \times 10^6$ Pa $\qquad$ −79.6 MPa

(b) $\sigma = -\frac{(7)(100 \times 10^3)}{\pi (0.040)^2} = -139.3 \times 10^6$ Pa $\qquad$ −139.3 MPa

(c) $\sigma = -\frac{(2.437)(100 \times 10^3)}{(0.040)^2} = -152.3 \times 10^6$ Pa $\qquad$ −152.3 MPa

Problem 4.107



4.107 A milling operation was used to remove a portion of a solid bar of square cross section. Knowing that $a = 30$ mm, $d = 20$ mm, and $\sigma_{all} = 60$ MPa, determine the magnitude P of the largest forces that can be safely applied at the centers of the ends of the bar.

$$A = ad, \qquad I = \frac{1}{12}ad^3, \qquad c = \frac{1}{2}d$$

$$e = \frac{a}{2} - \frac{d}{2}$$

$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{ad} + \frac{6Ped}{ad^3}$$

$$\sigma = \frac{P}{ad} + \frac{3P(a-d)}{ad^2} = KP \qquad \text{where } K = \frac{1}{ad} + \frac{3(a-d)}{ad^2}$$

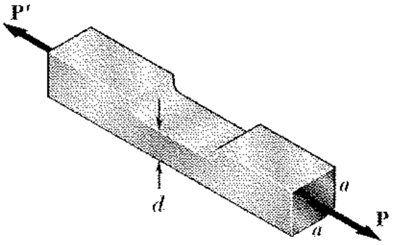
Data: $a = 30\text{ mm} = 0.030\text{ m} \qquad d = 20\text{ mm} = 0.020\text{ m}$

$$K = \frac{1}{(0.030)(0.020)} + \frac{(3)(0.010)}{(0.030)(0.020)^2} = 4.1667\times10^3\ \text{m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{60\times10^6}{4.1667\times10^3} = 14.40\times10^3\ \text{N}$$

$P = 14.40$ kN ◄

Problem 4.108



4.108 A milling operation was used to remove a portion of a solid bar of square cross section. Forces of magnitude $P = 18$ kN are applied at the centers of the ends of the bar. Knowing that $a = 30$ mm and $\sigma_{all} = 135$ MPa, determine the smallest allowable depth d of the milled portion of the bar.

$$A = ad, \qquad I = \frac{1}{12}ad^3, \qquad c = \frac{1}{2}d$$

$$e = \frac{a}{2} - \frac{d}{2}$$

$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{ad} + \frac{Pec}{I} = \frac{P}{ad} + \frac{P\frac{1}{2}(a-d)\frac{1}{2}d}{\frac{1}{12}ad^3} = \frac{P}{ad} + \frac{3P(a-d)}{ad^2}$$

$$\sigma = \frac{3P}{d^2} - \frac{2P}{ad} \qquad \text{or} \qquad \sigma d^2 + \frac{2P}{a}d - 3P = 0$$

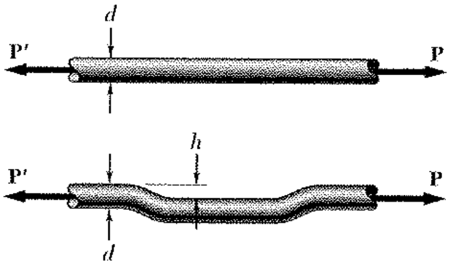
Solving for d, $$d = \frac{1}{2\sigma}\left\{\sqrt{\left(\frac{2P}{a}\right)^2 + 12P\sigma} - \frac{2P}{a}\right\}$$

Data: $a = 0.030\text{ m}, \qquad P = 18\times10^3\text{ N}, \qquad \sigma = 135\times10^6\text{ Pa}$

$$d = \frac{1}{(2)(135\times10^6)}\left\{\sqrt{\left[\frac{(2)(18\times10^3)}{0.030}\right]^2 + 12(18\times10^3)(135\times10^6)} - \frac{(2)(18\times10^3)}{0.030}\right\}$$
$$= 16.04\times10^{-3}$$

$d = 16.04$ mm ◄

Problem 4.109



4.109 An offset *h* must be introduced into a solid circular rod of diameter *d*. Knowing that the maximum stress after the offset is introduced must not exceed 5 times the stress in the rod when it is straight, determine the largest offset that can be used.

For centric loading $\sigma_c = \frac{P}{A}$

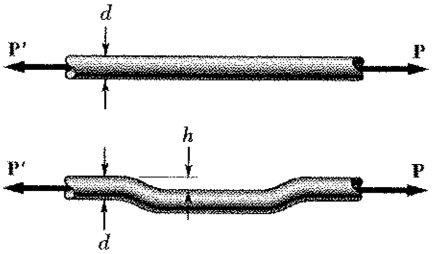
For eccentric loading $\sigma_e = \frac{P}{A} + \frac{Phc}{I}$

Given $\sigma_e = 5\sigma_c$

$$\frac{P}{A} + \frac{Phc}{I} = 5\frac{P}{A}$$

$$\frac{Phc}{I} = 4\frac{P}{A} \quad \therefore \quad h = \frac{4I}{cA} = \frac{(4(\frac{\pi}{64}d^4)}{(\frac{d}{2})(\frac{\pi}{4}d^2)} = \frac{1}{2}d = 0.500\,d \quad ◄$$

Problem 4.110



4.110 An offset *h* must be introduced into a metal tube of 18-mm outer diameter and 2-mm wall thickness. Knowing that the maximum stress after the offset is introduced must not exceed 4 times the stress in the rod when it is straight, determine the largest offset that can be used.

$c = \frac{1}{2}d = 9\,mm.$

$c_1 = c - t = 9 - 2 = 7\,mm$

$A = \pi(c^2 - c_1^2) = \pi(9^2 - 7^2)$
$= 100.5\,mm^2$

$I = \frac{\pi}{4}(c^4 - c_1^4) = \frac{\pi}{4}(9^4 - 7^4)$
$= 3267.3\,mm^4$

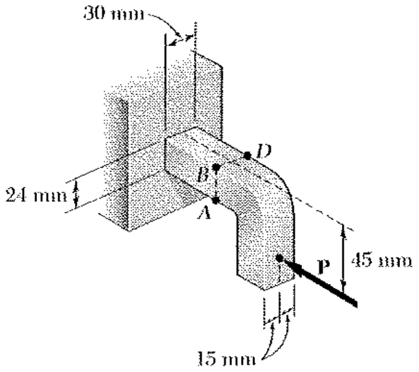
For centric loading $\sigma_{cen} = \frac{P}{A}$ For eccentric loading $\sigma_{ecc} = \frac{P}{A} + \frac{Phc}{I}$

$\sigma_{ecc} = 4\sigma_{cen}$ or $\frac{P}{A} + \frac{Phc}{I} = 4\frac{P}{A}$

$$\frac{hc}{I} = \frac{3}{A} \qquad h = \frac{3I}{Ac} = \frac{(3)(3267.3\times10^{-12})}{(100.5\times10^{-6})(0.009)} \qquad h = 10.8\,mm. ◄$$

Problem 4.111

4.111 Knowing that the allowable stress in section *ABD* is 70 MPa, determine the largest force **P** that can be applied to the bracket shown.



$$A = (0.03)(0.024) = 72\times10^{-5}\ m^2$$

$$I = \frac{1}{12}(0.03)(0.024)^3 = 34.56\times10^{-9}\ m^4$$

$$c = \frac{1}{2}(0.024) = 0.012\ m$$

$$e = 0.045 - \frac{0.024}{2} = 0.021\ m$$

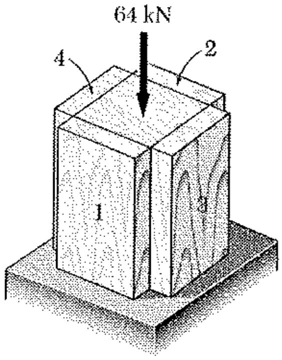
$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{Pec}{I} = PK$$

where $K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{72\times10^{-5}} + \frac{(0.021)(0.012)}{34.56\times10^{-9}} = 8680.6\ m^{-2}$

$$P = \frac{\sigma}{K} = \frac{70\times10^6}{8680.6}$$

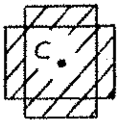
$P = 8.06\ kN$ ◀

Problem 4.112



4.112 A short column is made by nailing four 25 × 100 mm planks to a 100 × 100 mm timber. Determine the largest compressive stress created in the column by a 64-kN load applied as shown in the center of the top section of the timber if (*a*) the column is as described, (*b*) plank 1 is removed, (*c*) planks 1 and 2 are removed, (*d*) planks 1, 2, and 3 are removed, (*e*) all planks are removed.

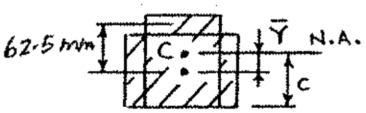
(a) Centric loading. $M = 0$ $\sigma = -\frac{P}{A}$



$A = (100)(100) + (4)(25)(100) = 2 \times 10^4 \text{ mm}^2$

$\sigma = -\frac{64 \times 10^3}{0.02} =$ $\sigma = -3.2 \text{MPa}$ ◄

(b) Eccentric loading. $M = Pe$ $\sigma = -\frac{P}{A} - \frac{Pec}{I}$



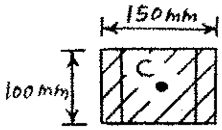
$A = (100)(100) + (3)(25)(100) = 17500 \text{ mm}^2$ $e = \bar{y}$

$\bar{y} = \frac{\Sigma A \bar{y}}{A} = \frac{(25)(100)(62.5)}{17500} = 8.93 \text{ mm}$

$I = \Sigma(\bar{I} + Ad^2) = \frac{1}{12}(150)(100)^3 + (150)(100)(8.93)^2 + \frac{1}{12}(100)(25)^3 + (100)(25)(53.75)^2$

$= 21.05 \times 10^6 \text{ mm}^4$

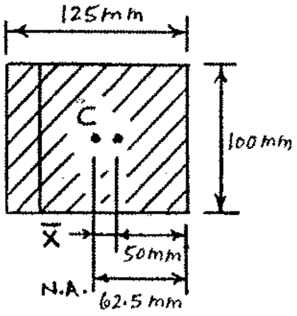
$\sigma = -\frac{64 \times 10^3}{17500 \times 10^{-6}} - \frac{(64 \times 10^3)(8.93 \times 10^{-3})(58.93 \times 10^{-3})}{21.05 \times 10^{-6}}$ $\sigma = -5.26 \text{ MPa}$ ◄



(c) Centric loading. $M = 0$ $\sigma = -\frac{P}{A}$

$A = (150)(100) = 15000 \text{ mm}^2$

$\sigma = -\frac{64 \times 10^3}{0.015}$ $\sigma = -4.3 \text{ MPa}$ ◄



(d) Eccentric loading. $M = Pe$ $\sigma = -\frac{P}{A} - \frac{Pec}{I}$

$A = (100)(100) + (1)(100)(25) = 12.5 \times 10^3 \text{ mm}^2$ $e = \bar{x}$

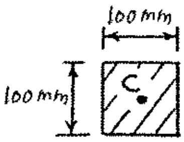
$\bar{x} = 62.5 - 50 = 12.5 \text{ mm}$

$I = \frac{1}{12}(100)(125)^3 = 16.276 \times 10^6 \text{ mm}^4$

$\sigma = -\frac{64 \times 10^3}{12.5 \times 10^{-3}} - \frac{(64 \times 10^3)(0.0125)(0.0625)}{16.276 \times 10^{-6}}$

$\sigma = -8.2 \text{ MPa}$ ◄

(e) Centric loading. $M = 0$ $\sigma = -\frac{P}{A}$

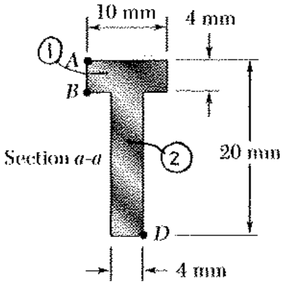


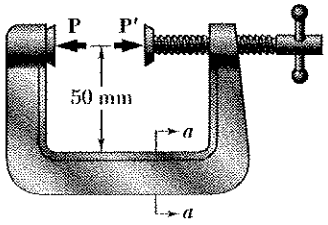
$A = (100)(100) = 1 \times 10^4 \text{ mm}^2$

$\sigma = -\frac{64 \times 10^3}{0.01}$ $\sigma = -6.4 \text{ MPa}$ ◄

Problem 4.113

4.113 Knowing that the clamp shown has been tightened on wooden planks being glued together until $P = 400$ N, determine in section *a-a* (*a*) the stress at point *A*, (*b*) the stress at point *D*, (*c*) the location of the neutral axis.





Locate centroid.

Part	A, mm^2	$\bar{y}_o, mm$	$A\bar{y}_o, mm^3$
①	40	18	720
②	64	8	512
	104		1232

$$\bar{Y}_o = \frac{1232}{104}$$
$$= 11.846 \text{ mm}$$

The centroid lies 11.846 mm above point D.

Eccentricity: $e = -(50 + 20 - 11.846) = -58.154$ mm

Bending couple: $M = Pe = (400)(-58.154 \times 10^{-3})$
$= -23.262$ N·m

$$A = 104 \text{ mm}^2 = 104 \times 10^{-6} \text{ m}^2$$
$$I_1 = \tfrac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \tfrac{1}{12}(10)(4)^3 + (40)(6.154)^2 = 1.5682 \times 10^3 \text{ mm}^4$$
$$I_2 = \tfrac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \tfrac{1}{12}(4)(16)^3 + (64)(3.846)^2 = 2.3120 \times 10^3 \text{ mm}^4$$
$$I = I_1 + I_2 = 3.8802 \times 10^3 \text{ mm}^4 = 3.8802 \times 10^{-9} \text{ m}^4$$

(a) <u>Stress at point A.</u> $y = 20 - 11.846 = 8.154 \text{ mm} = 8.154 \times 10^{-3}$ m

$$\sigma_A = \frac{P}{A} - \frac{My}{I} = \frac{400}{104 \times 10^{-6}} - \frac{(-23.262)(8.154 \times 10^{-3})}{3.8802 \times 10^{-9}} = 52.7 \times 10^6 \text{ Pa}$$

$\sigma_A = 52.7$ MPa ◄

(b) <u>Stress at point D.</u> $y = -11.846 \text{ mm} = -11.846 \times 10^{-3}$ m

$$\sigma_B = \frac{P}{A} + \frac{My}{I} = \frac{400}{104 \times 10^{-6}} - \frac{(-23.262)(-11.846 \times 10^{-3})}{3.8802 \times 10^{-9}} = -67.2 \times 10^6 \text{ Pa}$$

$\sigma_B = -67.2$ MPa ◄

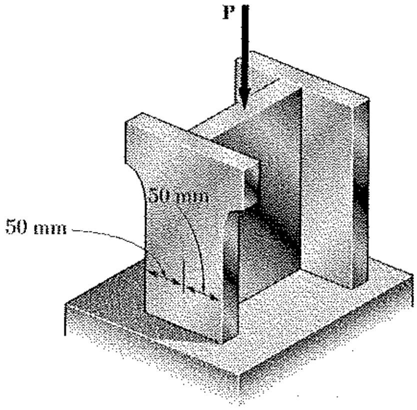
(c) <u>Location of neutral axis.</u> $\sigma = 0$

$$\sigma = \frac{P}{A} - \frac{My}{I} = \frac{P}{A} - \frac{Pey}{I} = 0$$

$$y = \frac{I}{Ae} = \frac{3.8802 \times 10^{-9}}{(104 \times 10^{-6})(-58.154 \times 10^{-3}} = -0.642 \times 10^{-6} \text{ m}$$
$$= -0.642 \text{ mm}$$

Neutral axis lies $11.846 - 0.642 = 11.205$ mm   11.20 mm above D ◄

Problem 4.114



4.114 Three steel plates, each of 25 × 150-mm cross section, are welded together to form a short H-shaped column. Later, for architectural reasons, a 25-mm strip is removed from each side of one of the flanges. Knowing that the load remains centric with respect to the original cross section, and that the allowable stress is 100 MPa, determine the largest force **P** (*a*) that could be applied to the original column, (*b*) that can be applied to the modified column.

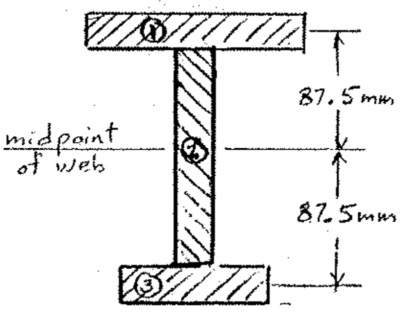
(a) Centric loading $\quad \sigma = -\frac{P}{A}$

$A = (3)(150)(25) = 11.25\times10^3\ mm^2 = 11.25\times10^{-3}\ m^2$

$P = -\sigma A = -(-100\times10^6)(11.25\times10^{-3})$

$= 1.125\times10^6\ N \qquad P = 1125\ kN$

(b) Eccentric loading (reduced cross section)



	$A, 10^3 mm^2$	$\bar{y}$, mm	$A\bar{y}$ $(10^3 mm^3)$	d, mm
①	3.75	87.5	328.125	76.5625
②	3.75	0	0	10.9375
③	2.50	-87.5	-218.75	98.4375
Σ	10.00		109.375	

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{109.375\times10^3}{10.00\times10^3} = 10.9375\ mm$$

The centroid lies 10.9375 mm from the midpoint of the web.

$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(150)(25)^3 + (3.75\times10^3)(76.5625)^2 = 22.177\times10^6\ mm^4$

$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(25)(150)^3 + (3.75\times10^3)(10.9375)^2 = 7.480\times10^6\ mm^4$

$I_3 = \frac{1}{12}b_3h_3^3 + A_3d_3^2 = \frac{1}{12}(100)(25)^3 + (2.50\times10^3)(98.4375)^2 = 24.355\times10^6\ mm^4$

$I = I_1 + I_2 + I_3 = 54.012\times10^6\ mm^4 = 54.012\times10^{-6}\ m^4$

$c = 10.9375 + 75 + 25 = 110.9375\ mm = 0.1109375\ m$

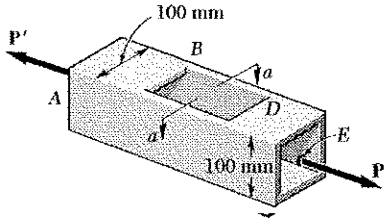
$M = Pe \quad$ where $e = 10.4375\ mm = 10.4375\times10^{-3}\ m$

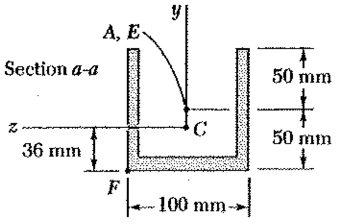
$\sigma = -\frac{P}{A} - \frac{Mc}{I} = -\frac{P}{A} - \frac{Pec}{I} = -KP \qquad A = 10.00\times10^{-3}\ m^2$

$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{10.00\times10^{-3}} + \frac{(10.9375\times10^{-3})(0.1109375)}{54.012\times10^{-6}} = 122.465\ m^{-2}$

$P = -\frac{\sigma}{K} = -\frac{(-100\times10^6)}{122.465} = 817\times10^3\ N \qquad P = 817\ kN$

Problem 4.115





4.115 In order to provide access to the interior of a hollow square tube of 6-mm wall thickness, the portion *CD* of one side of the tube has been removed. Knowing that the loading of the tube is equivalent to two equal and opposite 60-kN forces acting at the geometric centers *A* and *E* of the ends of the tube, determine (*a*) the maximum stress in section *a-a*, (*b*) the stress at point *F*. *Given:* the centroid of the cross section is at *C* and $I_z = 2 \times 10^6$ mm^4.

Area $A = (2)(100)(6) + (88)(6)$

$= 1728 \text{ mm}^2$

$M = Pe = (60)(0.05 - 0.036) = 0.84$ kNm.

(a) Maximum stress occurs at top of section where $y = 100 - 36 = 64$ mm.

$$\sigma_{max} = \frac{P}{A} + \frac{My}{I} = \frac{60 \times 10^3}{1728 \times 10^{-6}} + \frac{(840)(0.064)}{2 \times 10^{-6}}$$

$\sigma_{max} = 61.6$ MPa ◄

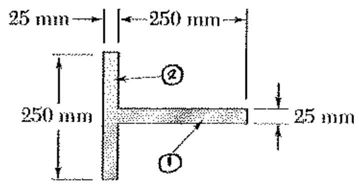
(b) At point F $y = -36$ mm.

$$\sigma_F = \frac{P}{A} + \frac{My}{I} = \frac{60 \times 10^3}{1728 \times 10^{-6}} + \frac{(840)(-0.036)}{2 \times 10^{-6}}$$

$\sigma_F = 19.6$ MPa. ◄

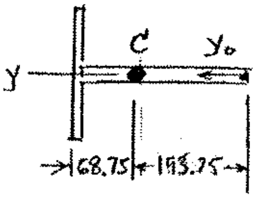
Problem 4.116

4.116 Knowing that the allowable stress in section *a-a* of the hydraulic press shown is 40 MPa in tension and 80 MPa in compression, determine the largest force **P** that can be exerted by the press.



Section *a-a*

Dimensions in mm

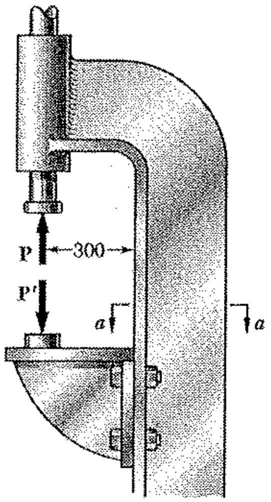


Locate centroid of cross section.

	A, mm²	$\bar{y}_0$, mm	$A\bar{y}_0$, mm³
①	6250	12.5	781.25×10^3
②	6250	262.5	1.640625×10^6
Σ	12500		2.421875×10^6

$$\bar{Y}_0 = \frac{2.421875 \times 10^3}{12500} = 193.75 \text{ mm}$$

The centroid lies at point C.



$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(25)(250)^3 + (6250)(68.75)^2$$
$$= 62.093 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A d_2^2 = \frac{1}{12}(250)(25)^3 + (6250)(68.75)^2$$
$$= 29.867 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 = 82.194 \times 10^6 \text{ mm}^4 = 91.960 \times 10^{-6} \text{ m}^4$$

$$A = 12500 \text{ mm}^2 = 12.5 \times 10^{-3} \text{ m}^2$$

Eccentricity $e = 300 + 25 + 250 - 193.75 = 381.25$ mm

The bending moment about the centroid is $M = -Pe$.

Stress: $\sigma = \frac{P}{A} - \frac{My}{I} = \frac{P}{A} + \frac{Pey}{I} = KP$ where $K = \frac{1}{A} + \frac{ey}{I}$

Then $P = \frac{\sigma}{K}$

Tensile stress: $\sigma = 40 \times 10^6$ Pa, $y = 81.25 \times 10^{-3}$ m

$$K = \frac{1}{12.5 \times 10^{-3}} + \frac{(381.25 \times 10^{-3})(81.25 \times 10^{-3})}{91.960 \times 10^{-6}} = 416.85 \text{ m}^{-2}$$

$$P = \frac{40 \times 10^6}{416.85} = 96.0 \times 10^3 \text{ N}$$

Compressive stress: $\sigma = -80 \times 10^6$ PA $y = -193.75 \times 10^{-3}$ m

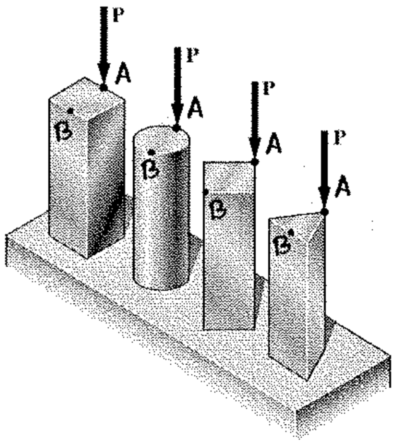
$$K = \frac{1}{12.5 \times 10^{-3}} + \frac{(381.25 \times 10^{-3})(-193.75 \times 10^{-3})}{91.960 \times 10^{-6}} = -723.25 \text{ m}^{-2}$$

$$P = \frac{-80 \times 10^6}{-723.25} = 110.6 \times 10^3 \text{ N}$$

Choose the smaller value $P = 96.0 \times 10^3$ N $P = 96.0$ kN ◀

Problem 4.117

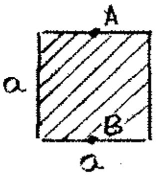
4.117 The four bars shown have the same cross-sectional area. For the given loadings, show that (*a*) the maximum compressive stresses are in the ratio 4:5:7:9, (*b*) the maximum tensile stresses are in the ratio 2:3:5:3. (*Note*: the cross section of the triangular bar is an equilateral triangle.)



Stresses

At A $\quad \sigma_A = -\frac{P}{A} - \frac{Pec_A}{I} = -\frac{P}{A}\left(1 + \frac{Aec_A}{I}\right)$

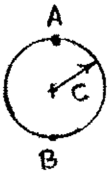
At B $\quad \sigma_B = -\frac{P}{A} + \frac{Pec_B}{I} = \frac{P}{A}\left(\frac{Aec_B}{I} - 1\right)$



$A_1 = a^2, \quad I_1 = \frac{1}{12}a^4, \quad C_A = C_B = \frac{1}{2}a, \quad e = \frac{1}{2}a$

$\sigma_A = -\frac{P}{A}\left(1 + \frac{(a^2)(\frac{1}{2}a)(\frac{1}{2}a)}{\frac{1}{12}a^2}\right) = -4\frac{P}{A_1}$

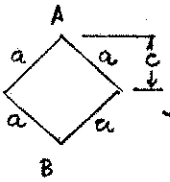
$\sigma_B = \frac{P}{A}\left(\frac{(a^2)(\frac{1}{2}a)(\frac{1}{2}a)}{\frac{1}{12}a^2} - 1\right) = 2\frac{P}{A_1}$



$A_2 = \pi c^2 = a^2 \quad \therefore c = \frac{a}{\sqrt{\pi}}, \quad I_2 = \frac{\pi}{4}c^4$

$\sigma_A = -\frac{P}{A_2}\left(1 + \frac{(\pi c^2)(c)(c)}{\frac{\pi}{4}c^4}\right) = -5\frac{P}{A_2}$

$\sigma_B = \frac{P}{A_2}\left(\frac{(\pi c^2)(c)(c)}{\frac{\pi}{4}c^4} - 1\right) = 3\frac{P}{A_2}$



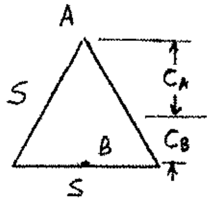
$A_3 = a^2 \qquad c = \frac{\sqrt{2}}{2}a \qquad I_3 = \frac{1}{12}a^4 \qquad e = c$

$\sigma_A = -\frac{P}{A_3}\left(1 + \frac{(a^2)(\frac{\sqrt{2}}{2}a)(\frac{\sqrt{2}}{2}a)}{\frac{1}{12}a^4}\right) = -7\frac{P}{A_3}$

$\sigma_B = \frac{P}{A_3}\left(\frac{(a^2)(\frac{\sqrt{2}}{2}a)(\frac{\sqrt{2}}{2}a)}{\frac{1}{12}a^4} - 1\right) = 5\frac{P}{A_3}$

$A_4 = \frac{1}{2}(s)(\frac{\sqrt{3}}{2}s) = \frac{\sqrt{3}}{4}s^2 \qquad I_4 = \frac{1}{36}s(\frac{\sqrt{3}}{2}s)^3 = \frac{\sqrt{3}}{96}s^4$

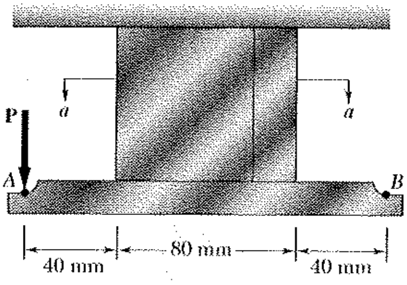
$C_A = \frac{2}{3}\frac{\sqrt{3}}{2}s = \frac{s}{\sqrt{3}} = e \qquad C_B = \frac{s}{2\sqrt{3}}$

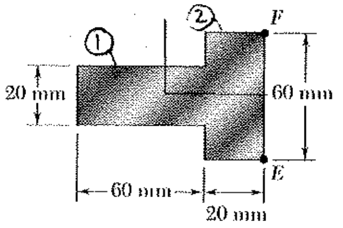


$\sigma_A = -\frac{P}{A_4}\left(1 + \frac{(\frac{\sqrt{3}}{4}s^2)(\frac{s}{\sqrt{3}})(\frac{s}{\sqrt{3}})}{\frac{\sqrt{3}}{96}s^4}\right) = -9\frac{P}{A_4}$

$\sigma_B = \frac{P}{A_4}\left(\frac{(\frac{\sqrt{3}}{4}s^2)(\frac{s}{\sqrt{3}})(\frac{s}{2\sqrt{3}})}{\frac{\sqrt{3}}{96}s^4} - 1\right) = 3\frac{P}{A_4}$

Problem 4.118





Section *a-a*

4.118 Knowing that the allowable stress is 150 MPa in section *a-a* of the hanger shown, determine (*a*) the largest vertical force **P** that can be applied at point *A*, (*b*) the corresponding location of the neutral axis of section *a-a*.

Locate centroid.

	A, mm²	$\bar{y}_o$, mm	$A\bar{y}_o$, mm³
①	1200	30	36×10^3
②	1200	70	84×10^3
Σ	2400		120×10^3

$$\bar{Y}_o = \frac{\Sigma A\bar{y}_o}{\Sigma A} = \frac{120\times10^3}{2400} = 50 \text{ mm}$$

The centroid lies 50 mm to the right of the left edge of the section.

Bending couple: $M = Pe$

$e = 40 + 50 = 90 \text{ mm} = 0.090 \text{ m}$

$$I_1 = \frac{1}{12}(20)(60)^3 + (1200)(20)^2 = 840\times10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12}(60)(20)^3 + (1200)(20)^2 = 520\times10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 1.360\times10^6 \text{ mm}^4 = 1.360\times10^{-6} \text{ m}^4, \qquad A = 2400\times10^{-6} \text{ m}^2$$

(a) Based on tensile stress at left edge: $y = -50 \text{ mm} = -0.050 \text{ m}$

$$\sigma = \frac{P}{A} - \frac{Pey}{I} = KP$$

$$K = \frac{1}{A} - \frac{ey}{I} = \frac{1}{2400\times10^{-6}} - \frac{(0.090)(-0.050)}{1.360\times10^{-6}} = 3.7255\times10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{150\times10^6}{3.7255\times10^3} = 40.3\times10^3 \text{ N} = 40.3 \text{ kN}$$ ◄

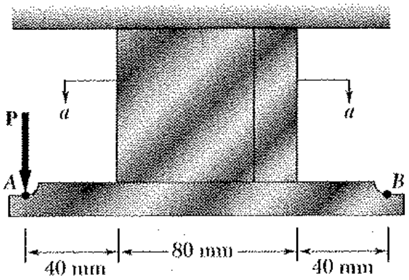
(b) Location of neutral axis: $\sigma = 0$

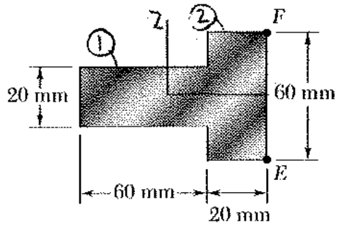
$$\sigma = \frac{P}{A} - \frac{Pey}{I} = 0 \qquad \frac{ey}{I} = \frac{1}{A}$$

$$y = \frac{I}{Ae} = \frac{1.360\times10^{-6}}{(2400\times10^{-6})(0.090)} = 6.30\times10^{-3} \text{ m} = 6.30 \text{ mm}$$

The neutral axis lies 6.30 mm to the right of the centroid or 56.30 mm from the left face. ◄

Problem 4.119





Section *a-a*

4.119 Solve Prob 4.118, assuming that the vertical force **P** is applied at point *B*.

4.118 Knowing that the allowable stress is 150 MPa in section *a-a* of the hanger shown, determine (*a*) the largest vertical force **P** that can be applied at point *A*, (*b*) the corresponding location of the neutral axis of section *a-a*.

Locate centroid.

	A, mm^2	$\bar{y}_o$, mm	$A\bar{y}_o$, mm^3
①	1200	30	36×10^3
②	1200	70	84×10^3
Σ	2400		120×10^3

$$\bar{Y}_o = \frac{\Sigma A\bar{y}_o}{\Sigma A} = \frac{120 \times 10^3}{2400} = 50 \text{ mm}$$

The centroid lies 50 mm to the right of the left edge of the section.

Bending couple: $M = Pe$

$$e = 50 - 120 = -70 \text{ mm} = -0.070 \text{ m}$$

$$I_1 = \tfrac{1}{12}(20)(60)^3 + (1200)(20)^2 = 840 \times 10^3 \text{ mm}^4$$

$$I_2 = \tfrac{1}{12}(60)(20)^3 + (1200)(20)^2 = 520 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 1.360 \times 10^6 \text{ mm}^4 = 1.360 \times 10^{-6} \text{ m}^4, \quad A = 2400 \times 10^{-6} \text{ m}^2$$

(a) Based on stress at left edge of section: $y = -50 \text{ mm} = -0.050 \text{ m}$

$$\sigma = \frac{P}{A} - \frac{Pey}{I} = K_L P$$

$$K_L = \frac{1}{A} - \frac{ey}{I} = \frac{1}{2400 \times 10^{-6}} - \frac{(-0.070)(-0.050)}{1.360 \times 10^{-6}} = -2.1569 \times 10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K_L} = \frac{-150 \times 10^6}{-2.1569 \times 10^3} = 69.6 \times 10^3 \text{ N}$$

Based on stress at right edge of section: $y = 30 \text{ mm} = 0.030 \text{ m}$

$$\sigma = \frac{P}{A} - \frac{Pey}{I} = K_R P$$

$$K_R = \frac{1}{A} - \frac{ey}{I} = \frac{1}{2400 \times 10^{-6}} - \frac{(-0.070)(0.030)}{1.360 \times 10^{-6}} = 1.9608 \times 10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{150 \times 10^6}{1.9608 \times 10^3} = 76.5 \times 10^3 \text{ N}$$

Choose the smaller value $P = 69.6 \times 10^3 \text{ N} = 69.6 \text{ kN}$ ◀

(b) Location of neutral axis: $\sigma = 0$

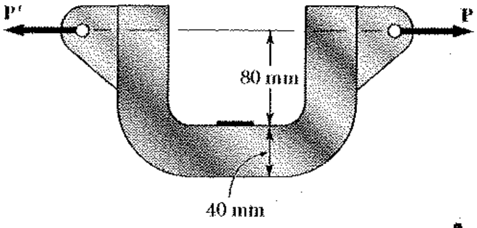
$$\sigma = \frac{P}{A} - \frac{Pey}{I} = 0 \qquad \frac{ey}{I} = \frac{1}{A}$$

$$y = \frac{I}{Ae} = \frac{1.360 \times 10^{-6}}{(2400 \times 10^{-6})(-0.070)} = -8.10 \times 10^{-3} \text{m} = -8.10 \text{ mm}$$

Neutral axis lies $50 - 8.10 = 41.9$ mm from left face. ◀

Problem 4.120

4.120 The C-shaped steel bar is used as a dynamometer to determine the magnitude P of the forces shown. Knowing that the cross section of the bar is a square of side 40 mm and the strain on the inner edge was measured and found to be 450 μ, determine the magnitude P of the forces. Use $E = 200$ GPa.



At the strain gage location

$$\sigma = E\varepsilon = (200\times10^{9})(450\times10^{-6})$$
$$= 90\ \text{MPa}$$

$A = (40)(40) = 1600\ \text{mm}^2$

$I = \frac{1}{12}(40)(40)^3 = 213333.3\ \text{mm}^4$

$e = 80+20 = 100\ \text{mm}.$

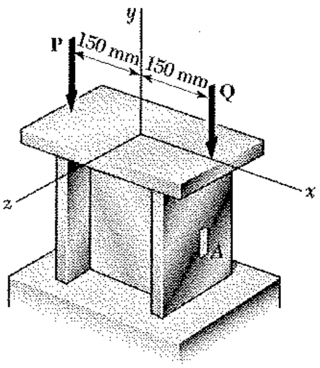
$c = 20\ \text{mm}.$

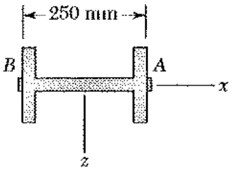
$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{Pec}{I} = KP$$

$$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{1600\times10^{-6}} + \frac{(0.1)(0.02)}{213333.3\times10^{-12}} = 10000\ \text{m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{90000\times10^{3}}{10000} = 9000\text{N} \qquad P = 9\text{kN}$$

Problem 4.121





$A = 6450 \text{ mm}^2$
$I_z = 114 \times 10^6 \text{ mm}^4$

4.121 A short length of a rolled-steel column supports a rigid plate on which two loads **P** and **Q** are applied as shown. The strains at two points *A* and *B* on the center line of the outer faces of the flanges have been measured and found to be

$$\epsilon_A = -400 \times 10^{-6} \text{ mm/mm} \qquad \epsilon_B = -300 \times 10^{-6} \text{ mm/mm}$$

Knowing that $E = 200$ GPa, determine the magnitude of each load.

Stresses at A and B from strain gages

$$\sigma_A = E\varepsilon_A = (200\times10^9)(-400\times10^{-6}) = -80 \text{ MPa.}$$

$$\sigma_B = E\,\varepsilon_B = (200\times10^9)(-300\times10^{-6}) = -60 \text{ MPa.}$$

Centric force $F = P + Q$

Bending couple $M = 6P - 6Q$

$c = 125$ mm.

$$\sigma_A = -\frac{F}{A} + \frac{Mc}{I} = -\frac{P+Q}{6450\times10^{-6}} + \frac{(0.15P - 0.15Q)(0.125)}{114\times10^{-6}}$$

$$-80\times10^6 = 9.5P - 319.5Q. \qquad (1)$$

$$\sigma_B = -\frac{F}{A} - \frac{Mc}{I} = -\frac{P+Q}{6450\times10^{-6}} - \frac{(0.15P - 0.15Q)(0.125)}{114\times10^{-6}.}$$

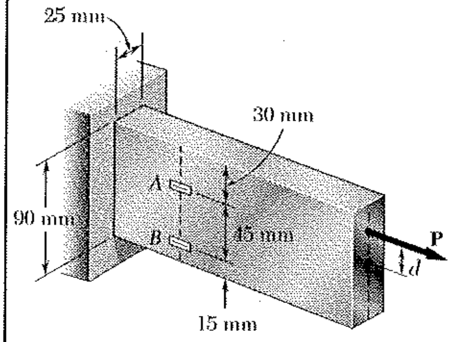
$$-60\times10^6 = -319.5P + 9.5Q \qquad (2)$$

Solving (1) and (2) simultaneously

$P = 195.4$ kN

$Q = 256.2$ kN

Problem 4.122



4.122 An eccentric force **P** is applied as shown to a steel bar of 25 × 90-mm cross section. The strains at A and B have been measured and found to be

$$\varepsilon_A = +350\ \mu \qquad \varepsilon_B = -70\ \mu$$

Knowing that $E = 200$ GPa, determine (a) the distance d, (b) the magnitude of the force **P**.

$h = 15 + 45 + 30 = 90$ mm

$b = 25$ mm $\qquad c = \frac{1}{2}h = 45\text{mm} = 0.045$ m

$A = bh = (25)(90) = 2.25\times10^{3}\ \text{mm}^2 = 2.25\times10^{-3}\ \text{m}^2$

$I = \frac{1}{12}bh^3 = \frac{1}{12}(25)(90)^3 = 1.51875\times10^{6}\ \text{mm}^4$

$= 1.51875\times10^{-6}\ \text{m}^4$

$y_A = 60 - 45 = 15\ \text{mm} = 0.015\text{m}$, $\quad y_B = 15 - 45 = -30\ \text{mm} = -0.030$ m

Stresses from strain gages at A and B;

$\sigma_A = E\varepsilon_A = (200\times10^{9})(350\times10^{-6}) = 70\times10^{6}$ Pa

$\sigma_B = E\varepsilon_B = (200\times10^{9})(-70\times10^{-6}) = -14\times10^{6}$ Pa

$$\sigma_A = \frac{P}{A} - \frac{My_A}{I} \qquad (1)$$

$$\sigma_B = \frac{P}{A} - \frac{My_B}{I} \qquad (2)$$

Subtracting, $\quad \sigma_A - \sigma_B = -\dfrac{M(y_A - y_B)}{I}$

$$M = -\frac{I(\sigma_A - \sigma_B)}{y_A - y_B} = -\frac{(1.51875\times10^{-6})(84\times10^{6})}{0.045} = -2835\ \text{N}\cdot\text{m}$$

Multiplying (2) by y_A and (1) by y_B and subtracting,

$$y_A\sigma_B - y_B\sigma_A = (y_A - y_B)\frac{P}{A}$$

$$P = \frac{A(y_A\sigma_B - y_B\sigma_A)}{y_A - y_B} = \frac{(2.25\times10^{-3})\left[(0.015)(-14\times10^{6}) - (-0.030)(70\times10^{6})\right]}{0.045}$$

$= 94.5\times10^{3}$ N

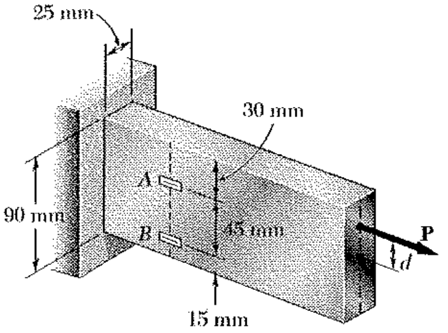
(a) $\quad M = -Pd \therefore \quad d = -\dfrac{M}{P} = -\dfrac{-2835}{94.5\times10^{3}} = 0.030\ \text{m} \qquad d = 30.0\ \text{mm}$ ◄

(b) $\qquad P = 94.5$ kN ◄

Problem 4.123

4.123 Solve prob. 4.122, assuming that the measured strains are

$\epsilon_A = +600\,\mu$ $\qquad$ $\epsilon_B = +420\,\mu$



4.122 An eccentric force **P** is applied as shown to a steel bar of 25 × 90-mm cross section. The strains at *A* and *B* have been measured and found to be

$\epsilon_A = +350\,\mu$ $\qquad$ $\epsilon_B = -70\,\mu$

Knowing that $E = 200$ GPa, determine (*a*) the distance *d*, (*b*) the magnitude of the force **P**.

$h = 15 + 45 + 30 = 90 \text{ mm}$

$b = 25 \text{ mm} \qquad c = \frac{1}{2}h = 45 \text{ mm} = 0.045 \text{ m}$

$A = bh = (25)(90) = 2.25 \times 10^3 \text{ mm}^2 = 2.25 \times 10^{-3} \text{ m}^2$

$I = \frac{1}{12}bh^3 = \frac{1}{12}(25)(90)^3 = 1.51875 \times 10^6 \text{ mm}^4 = 1.51875 \times 10^{-6} \text{ m}^4$

$y_A = 60 - 45 = 15 \text{ mm} = 0.015 \text{ m}, \qquad y_B = 15 - 45 = -30 \text{ mm} = -0.030 \text{ m}$

Stresses from strain gages at A and B:

$\sigma_A = E\epsilon_A = (200 \times 10^9)(600 \times 10^{-6}) = 120 \times 10^6 \text{ Pa}$

$\sigma_B = E\epsilon_B = (200 \times 10^9)(420 \times 10^{-6}) = 84 \times 10^6 \text{ Pa}$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{I} \qquad (1)$$

$$\sigma_B = \frac{P}{A} - \frac{My_B}{I} \qquad (2)$$

Subtracting, $\quad \sigma_A - \sigma_B = -\frac{M(y_A - y_B)}{I}$

$$M = -\frac{I(\sigma_A - \sigma_B)}{y_A - y_B} = -\frac{(1.51875 \times 10^{-6})(36 \times 10^6)}{0.045} = -1215 \text{ N·m}$$

Multiplying (2) by y_A and (1) by y_B and subtracting,

$$y_A\sigma_B - y_B\sigma_A = (y_A - y_B)\frac{P}{A}$$

$$P = \frac{A(y_A\sigma_B - y_B\sigma_A)}{y_A - y_B} = \frac{(2.25 \times 10^{-3})[(0.015)(84 \times 10^6) - (-0.030)(120 \times 10^6)]}{0.045}$$

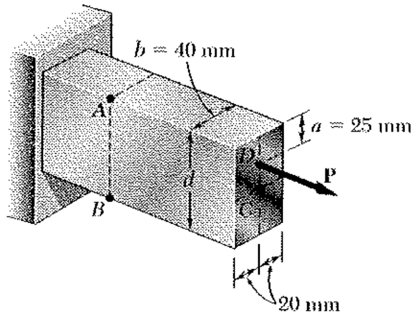
$= 243 \times 10^3 \text{ N}$

$M = -Pd$

(a) $\quad \therefore d = -\frac{M}{P} = -\frac{-1215}{243 \times 10^3} = 5 \times 10^{-3} \text{ m} \qquad d = 5.00 \text{ mm}$ ◄

(b) $\qquad P = 243 \text{ kN}$ ◄

Problem 4.124



4.124 The eccentric axial force **P** acts at point *D*, which must be located 25 mm below the top surface of the steel bar shown. For *P* = 60 kN, determine (*a*) the depth *d* of the bar for which the tensile stress at point *A* is maximum, (*b*) the corresponding stress at point *A*.

$A = bd \qquad I = \frac{1}{12}bd^3$

$c = \frac{1}{2}d \qquad e = \frac{1}{2}d - a$

$\sigma_A = \frac{P}{A} + \frac{Pec}{I}$

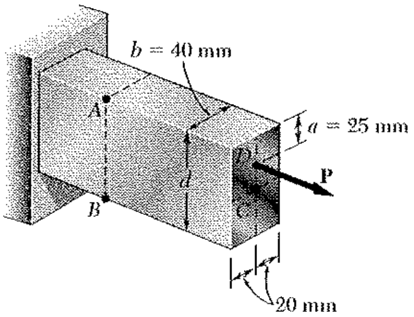
$\sigma_A = \frac{P}{b}\left\{\frac{1}{d} + \frac{12(\frac{1}{2}d - a)(\frac{1}{2}d)}{d^3}\right\} = \frac{P}{b}\left\{\frac{4}{d} - \frac{6a}{d^2}\right\}$

(a) Depth d for maximum σ_A. Differentiate with respect to d.

$\frac{d\sigma_A}{dd} = \frac{P}{b}\left\{-\frac{4}{d^2} + \frac{12a}{d^3}\right\} = 0 \qquad d = 3a = 75 \text{ mm}$ ◄

(b) $\sigma_A = \frac{60\times10^3}{40\times10^{-3}}\left\{\frac{4}{75\times10^{-3}} - \frac{(6)(25\times10^{-3})}{(75\times10^{-3})^2}\right\} = 40\times10^6 \text{ Pa} = 40 \text{ MPa}$ ◄

Problem 4.125



4.125 For the bar and loading of Prob. 4.124, determine (*a*) the depth *d* of the bar for which the compressive stress at point *B* is maximum, (*b*) the corresponding stress at point *B*.

4.124 The eccentric axial force **P** acts at point *D*, which must be located 25 mm below the top surface of the steel bar shown. For *P* = 60 kN, determine (*a*) the depth *d* of the bar for which the tensile stress at point *A* is maximum, (*b*) the corresponding stress at point *A*.

$A = bd \qquad I = \frac{1}{12}bd^3$

$c = \frac{1}{2}d \qquad e = \frac{1}{2}d - a$

$\sigma_B = \frac{P}{A} - \frac{Pec}{I}$

$\sigma_B = \frac{P}{b}\left\{\frac{1}{d} - \frac{(12)(\frac{1}{2}d - a)(\frac{1}{2}d)}{d^2}\right\} = \frac{P}{b}\left\{-\frac{2}{d} + \frac{6a}{d^2}\right\}$

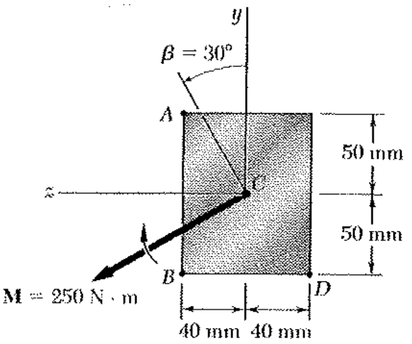
(a) Depth d for maximum σ_B: Differentiate with respect to d.

$\frac{d\sigma_B}{dd} = \frac{P}{b}\left\{\frac{2}{d^2} - \frac{12a}{d^3}\right\} = 0 \qquad d = 6a = 150 \text{ mm}$ ◄

(b) $\sigma_B = \frac{60\times10^3}{40\times10^{-3}}\left\{-\frac{2}{150\times10^{-3}} + \frac{(6)(25\times10^{-3})}{(150\times10^{-3})^2}\right\} = -10\times10^6 \text{ Pa} = -10 \text{ MPa}$ ◄

Problem 4.126

4.126 through *4.128* The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.



$$I_z = \frac{1}{12}(80)(100)^3 = 6.6667\times10^{6}\ \text{mm}^4 = 6.6667\times10^{-6}\ \text{m}^4$$

$$I_y = \frac{1}{12}(100)(80)^3 = 4.2667\times10^{6}\ \text{mm}^4 = 4.2667\times10^{-6}\ \text{m}^4$$

$$y_A = -y_B = -y_D = 50\ \text{mm}$$

$$z_A = z_B = -z_D = 40\ \text{mm}$$

$$M_y = -250\sin 30^\circ = -125\ \text{N}\cdot\text{m} \qquad M_z = 250\cos 30^\circ = 216.51\ \text{N}\cdot\text{m}$$

(a) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(216.51)(0.050)}{6.6667\times10^{-6}} + \frac{(-125)(0.040)}{4.2667\times10^{-6}}$$

$$= -2.80\times10^{6}\ \text{Pa} \qquad -2.80\ \text{MPa} ◄$$

(b) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(216.51)(-0.050)}{6.6667\times10^{-6}} + \frac{(-125)(0.040)}{4.2667\times10^{-6}}$$

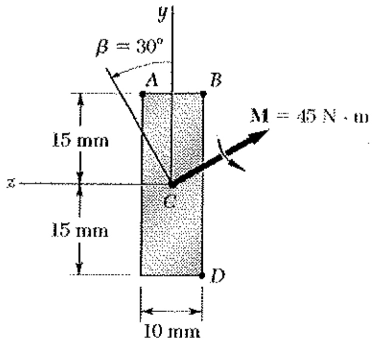
$$= 0.452\times10^{3}\ \text{Pa} \qquad 0.452\ \text{MPa} ◄$$

(c) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(216.51)(-0.050)}{6.6667\times10^{-6}} + \frac{(-125)(-0.040)}{4.2667\times10^{-6}}$$

$$= 2.80\times10^{6}\ \text{Pa} \qquad 2.80\ \text{MPa} ◄$$

Problem 4.127

4.126 through 4.128 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.



$I_z = \frac{1}{12}(10)(30)^3 = 22500 \text{ mm}^4$

$I_y = \frac{1}{12}(30)(10)^3 = 2500 \text{ mm}^4.$

$y_A = y_B = -y_D = 15 \text{ mm}$

$z_A = -z_B = -z_D = (\frac{1}{2})(10) = 5 \text{ mm}.$

$M_y = 4.5 \cos 60° = 22.5 \text{ Nm}, \qquad M_z = -4.5 \sin 60° = -39 \text{ Nm}.$

(a) $\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(-39)(0.015)}{22500 \times 10^{-12}} + \frac{(22.5)(0.005)}{2500 \times 10^{-12}}$

$= 71 \text{ MPa}$ ◄

(b) $\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(-39)(0.015)}{22500 \times 10^{-12}} + \frac{(22.5)(-0.005)}{2500 \times 10^{-12}}$

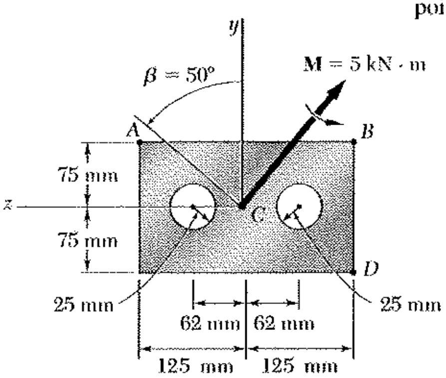
$= -19 \text{ MPa}$ ◄

(c) $\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(-39)(0.015)}{22500 \times 10^{-12}} + \frac{(22.5)(-0.005)}{2500 \times 10^{-12}}$

$= -71 \text{ MPa}$ ◄

Problem 4.128

4.126 through *4.128* The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.



$M_z = -5 \sin 40° = -3.214$ kNm

$M_y = 5 \cos 40° = 3.83$ kNm

$y_A = y_B = -y_D = 75$ mm

$z_A = -z_B = -z_D = 125$ mm

$$I_z = \frac{1}{12}(250)(150)^3 - 2\left[\frac{\pi}{4}(25)^4\right] = 69.7 \times 10^6 \text{ mm}^4$$

$$I_y = \frac{1}{12}(150)(250)^3 - 2\left[\frac{\pi}{4}(25)^4 + \pi(25)^2(62)^2\right] = 187.2 \times 10^6 \text{ mm}^4$$

(a) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(-3214)(0.075)}{69.7 \times 10^{-6}} + \frac{(3830)(0.125)}{187.2 \times 10^{-6}}$$

$= 6.016$ MPa $\qquad \sigma_A = 6.02$ MPa ◄

(b) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(-3214)(0.075)}{69.7 \times 10^{6}} + \frac{(3830)(-0.125)}{187.2 \times 10^{-6}}$$

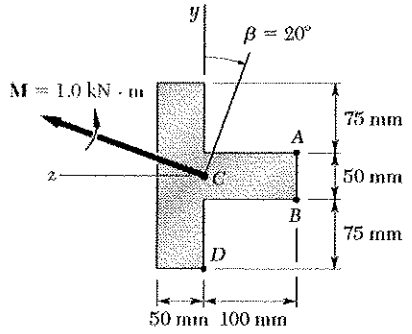
$= 0.9$ MPa $\qquad \sigma_B = 0.9$ MPa ◄

(c) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(-3214)(-0.075)}{69.7 \times 10^{-6}} + \frac{(3830)(-0.125)}{187.2 \times 10^{-6}}$$

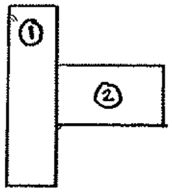
$= -6.016$ MPa $\qquad \sigma_D = -6.02$ MPa ◄

Problem 4.129

4.129 through 4.131 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.



Locate centroid



	A, mm^2	$\bar{z}, mm.$	$A\bar{z}, mm^3.$
①	10000	-25	-250 000
②	5000	50	250 000
Σ	15000		0

The centroid lies at point C

$$I_z = \tfrac{1}{12}(50)(200)^3 + \tfrac{1}{12}(100)(50)^3 = 34.375 \times 10^6 \; mm^4.$$

$$I_y = \tfrac{1}{3}(200)(50)^3 + \tfrac{1}{3}(50)(100)^3 = 25 \times 10^6 \; mm^4.$$

$y_A = -y_B = 25 \, mm,\qquad y_D = -100 \, mm.$

$z_A = z_B = -100 \, mm, \qquad z_D = 0$

$M_z = 1 \cos 20^\circ = 0.94 \; kNm.$

$M_y = 1 \sin 20^\circ = 0.342 \; kNm$

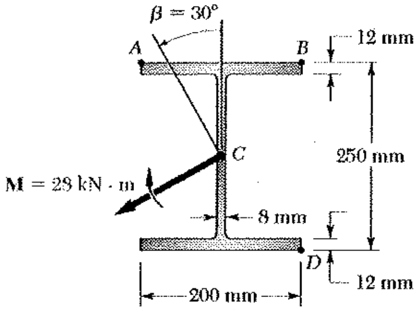
(a) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(940)(0.025)}{34.375 \times 10^{-6}} + \frac{(342)(-0.1)}{25 \times 10^{-6}} = 2.05 \, MPa.$$ ◄

(b) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(940)(-0.025)}{34.375 \times 10^{-6}} + \frac{(342)(-0.1)}{25 \times 10^{-6}} = -0.684 \, MPa$$ ◄

(c) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(940)(-0.1)}{34.375 \times 10^{-6}} + \frac{(342)(0)}{25 \times 10^{-6}} = 2.73 \, MPa.$$ ◄

Problem 4.130

4.129 through 4.131 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.



Flange: $I_z = \frac{1}{12}(200)(12)^3 + (200)(12)(119)^2$

$= 34015200 \text{ mm}^4.$

$I_y = \frac{1}{12}(12)(200)^3 = 8\times10^6 \text{ mm}^4.$

Web: $I_z = \frac{1}{12}(8)(226)^3 = 7695451 \text{ mm}^4$

$I_y = \frac{1}{12}(226)(8)^3 = 9643 \text{ mm}^4.$

Total: $I_z = (2)(34015200) + 7695451 = 75725851 \text{ mm}^4$

$I_y = (2)(8\times10^6) + 9643 = 16009643 \text{ mm}^4$

$y_A = y_B = -y_D = 125 \text{ mm}; \quad z_A = -z_B = -z_c = 100 \text{ mm}.$

$M_z = 28 \cos 30° = 24.25 \text{ kNm}.$

$M_y = -28 \sin 30° = -14 \text{ kNm}$

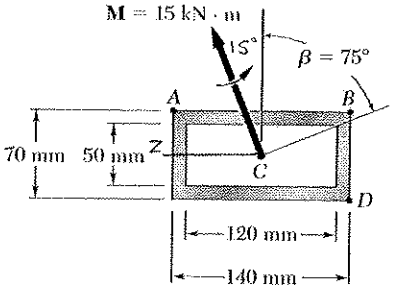
(a) $\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(24.25\times10^3)(0.125)}{75725851\times10^{-12}} + \frac{(-14\times10^3)(0.1)}{16009643\times10^{-12}} = -127.5 \text{ MPa}$ ◀

(b) $\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(24250)(0.125)}{75725851\times10^{-12}} + \frac{(-14000)(-0.1)}{16009643\times10^{-12}} = 47.4 \text{ MPa}.$ ◀

(c) $\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(24250)(-0.125)}{75725851\times10^{-12}} + \frac{(-14000)(-0.1)}{16009643\times10^{-12}} = 127.5 \text{ MPa}.$ ◀

Problem 4.131

4.129 through 4.131 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.



$$I_z = \frac{1}{12}(140)(70)^3 - \frac{1}{12}(120)(50)^3 = 2.7517\times10^6 \text{ mm}^4$$
$$= 2.7517\times10^{-6} \text{ m}^4$$
$$I_y = \frac{1}{12}(70)(140)^3 - \frac{1}{12}(50)(120)^3 = 8.8067\times10^6 \text{ mm}^4$$
$$= 8.8067\times10^{-6} \text{ m}^4$$
$$y_A = y_B = -y_D = 35 \text{ mm} = 0.035 \text{ m}$$
$$z_A = -z_B = -z_D = 70 \text{ mm} = 0.070 \text{ m}$$
$$M_z = M\sin 15^\circ = (15\times10^3)\sin 15^\circ = 3.8823\times10^3 \text{ N}\cdot\text{m}$$
$$M_y = M\cos 15^\circ = (15\times10^3)\cos 15^\circ = 14.4889\times10^3 \text{ N}\cdot\text{m}$$

(a) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(3.8823\times10^3)(0.035)}{2.7517\times10^{-6}} + \frac{(14.4889\times10^3)(0.070)}{8.8067\times10^{-6}}$$
$$= 65.8\times10^6 \text{ Pa}$$

$\sigma_A = 65.8$ MPa

(b) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(3.8823\times10^3)(0.035)}{2.7517\times10^{-6}} + \frac{(14.4889\times10^3)(-0.070)}{8.8067\times10^{-6}}$$
$$= -164.5\times10^6 \text{ Pa}$$

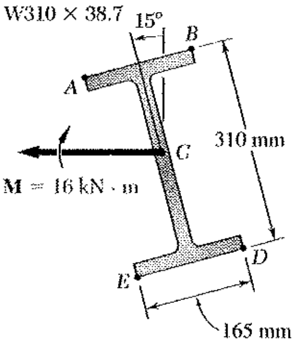
$\sigma_B = -164.5$ MPa

(c) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(3.8823\times10^3)(-0.035)}{2.7517\times10^{-6}} + \frac{(14.4889\times10^3)(-0.070)}{8.8067\times10^{-6}}$$
$$= -65.8\times10^6 \text{ Pa}$$

$\sigma_D = -65.8$ MPa

Problem 4.132

4.132 The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.



For W 310 × 38.7 rolled steel shape,

$I_z = 85.1 \times 10^6 \text{ mm}^4 = 85.1 \times 10^{-6} \text{ m}^4$

$I_y = 7.27 \times 10^6 \text{ mm}^4 = 7.27 \times 10^{-6} \text{ m}^4$

$y_A = y_B = -y_D = -y_E = (\frac{1}{2})(310) = 155 \text{ mm}$

$z_A = z_E = -z_B = -z_D = (\frac{1}{2})(165) = 82.5 \text{ mm}$

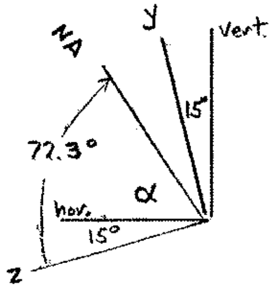
$M_z = (16 \times 10^3) \cos 15° = 15.455 \times 10^3 \text{ N} \cdot \text{m}$

$M_y = (16 \times 10^3) \sin 15° = 4.1411 \times 10^3 \text{ N} \cdot \text{m}$

(a) $\tan \varphi = \frac{I_z}{I} \tan \theta = \frac{85.1 \times 10^{-6}}{7.27 \times 10^{-6}} \tan 15° = 3.1365$

$\varphi = 72.3°$

$\alpha - 72.3 - 15 = 57.3°$ ◄



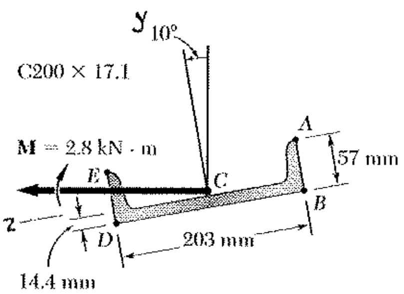
(b) Maximum tensile stress occurs at point E.

$$\sigma_E = -\frac{M_z y_E}{I_z} + \frac{M_y z_E}{I_y} = -\frac{(15.455 \times 10^3)(-155 \times 10^{-3})}{85.1 \times 10^{-6}} + \frac{(4.1411 \times 10^3)(82.5 \times 10^{-3})}{7.27 \times 10^{-6}}$$

$= 75.1 \times 10^6 \text{ Pa}$ $\sigma_E = 75.1 \text{ MPa}$ ◄

Problem 4.133

4.133 and 4.134 The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.



For C200 × 17.1 rolled steel shape,

$I_z = 0.538 \times 10^6 \text{ mm}^4 = 0.538 \times 10^{-6} \text{ m}^4$

$I_y = 13.4 \times 10^6 \text{ mm}^4 = 13.4 \times 10^{-6} \text{ m}^4$

$z_E = z_D = -z_A = -z_B = \frac{1}{2}(203) = 101.5 \text{ mm}$

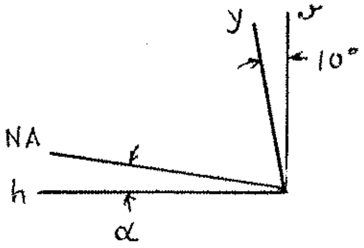
$y_D = y_B = -14.4 \text{ mm}$

$y_E = y_A = 57 - 14.4 = 42.6 \text{ mm}$

$M_z = (2.8 \times 10^3) \cos 10° = 2.7575 \times 10^3 \text{ N·m}$

$M_y = (2.8 \times 10^3) \sin 10° = 486.21 \text{ N·m}$

(a) $\tan \varphi = \frac{I_z}{I_y} \tan \theta = \frac{0.538}{13.4} \tan 10° = 0.007079$



$\varphi = 0.4056°$

$\alpha = 10° - 0.4056°$

$\alpha = 9.59°$ ◄

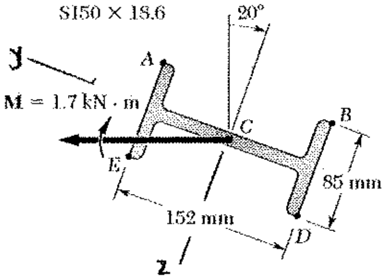
(b) Maximum tensile stress occurs at point D.

$$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(2.7575 10^3)(-14.4 \times 10^{-3})}{0.538 \times 10^{-6}} + \frac{(486.21)(0.1015)}{13.4 \times 10^{-6}}$$

$$= 73.807 \times 10^6 + 3.682 \times 10^6 = 77.5 \times 10^6 \text{ Pa}$$

$\sigma_D = 77.5 \text{ MPa}$ ◄

Problem 4.134



4.133 and 4.134 The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.

For S150X 18.6 rolled steel shape

$I_z = 9.11 \times 10^6 \text{ mm}^4$

$I_y = 0.782 \times 10^6 \text{ mm}^4$

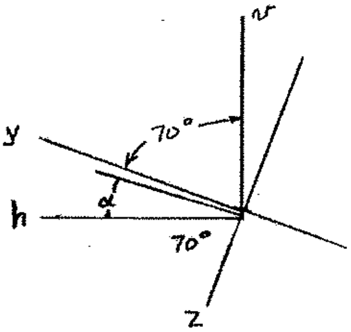
$z_E = -z_A = -z_B = z_D = \frac{1}{2}(85) = 42.5 \text{ mm}$

$y_A = y_B = -y_D = -y_E = \frac{1}{2}(152) = 76 \text{ mm}$

$M_z = 1.7 \sin 20° = 0.581 \text{ kNm}$

$M_y = 1.7 \cos 20° = 1.6 \text{ kNm}$

(a) $\tan \varphi = \frac{I_z}{I_y} \tan \theta = \frac{9.11}{0.782} \tan(90° - 20°) = 32.0$



$\varphi = 88.2°$

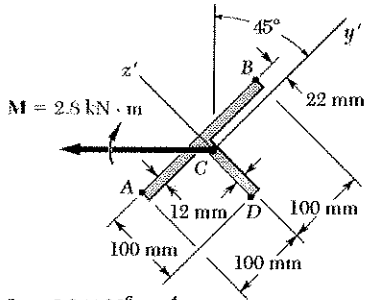
$\alpha = 88.2° - 70° = 18.2°$

(b) Maximum tensile stress occurs at point D.

$$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(581)(-0.076)}{9.11 \times 10^{-6}} + \frac{(1600)(0.0425)}{0.782 \times 10^{-6}} = 91.6 \text{ MPa}$$

Problem 4.135

4.135 and 4.136 The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.



$I_{y'} = 2.8 \times 10^6\ \text{mm}^4$
$I_{z'} = 8.9 \times 10^6\ \text{mm}^4$

$I_{z'} = 8.9 \times 10^6\ mm^4,\quad I_{y'} = 2.8 \times 10^6\ mm^6$

$z_A' = z_B' = 22\ mm.$

$z_D = -100 + 22 = -78$

$y_A = -100\ m,\ y_B = 100\ m,\ y_D = -12\ mm$

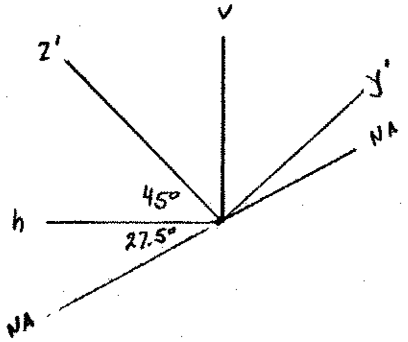
$M_{y'} = -2.8 \sin 45° = -1.98\ kN m.$

$M_{z'} = 2.8 \cos 45° = 1.98\ kN m.$

(a) $\tan\varphi = \frac{I_{z'}}{I_{y'}} \tan\theta = \frac{8.9}{2.8}\tan(-45°) = -3.1786.$

$\varphi = -72.5°$

$\alpha = 72.5° - 45° = 27.5°$



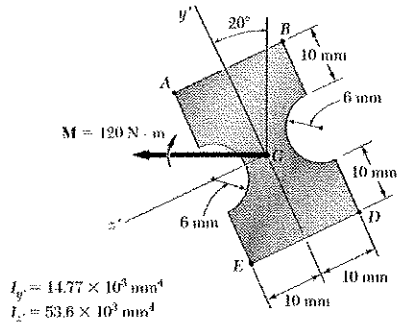
(b) Maximum tensile stress occurs at point D.

$$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(1980)(-0.012)}{8.9 \times 10^{-6}} + \frac{(-1980)(-0.078)}{2.8 \times 10^{-6}}$$

$$= 57.8\ MPa$$

Problem 4.136

4.135 and 4.136 The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.

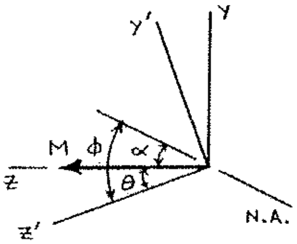


$$I_{z'} = 53.6 \times 10^3 \text{ mm}^4 = 53.6 \times 10^{-9} \text{ m}^4$$

$$I_{y'} = 14.77 \times 10^3 \text{ mm}^4 = 14.77 \times 10^{-9} \text{ m}^4$$

$$M_{z'} = 120 \sin 70° = 112.763 \text{ N·m}$$

$$M_{y'} = 120 \cos 70° = 41.042 \text{ N·m}$$



(a) $\theta = 20°$.

$$\tan\varphi = \frac{I_{z'}}{I_{y'}} \tan\theta = \frac{53.6 \times 10^{-9}}{14.77 \times 10^{-9}} \tan 20°$$

$$= 1.32054$$

$$\varphi = 52.871°$$

$$\alpha = 52.871° - 20° \qquad \alpha = 32.9°$$ ◄

(b) The maximum tensile stress occurs at point E.

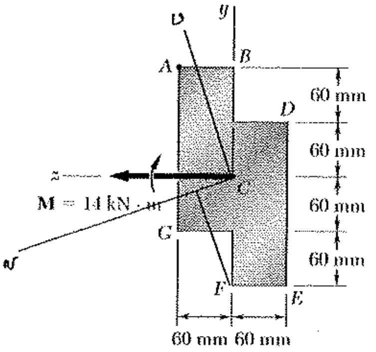
$$y'_E = -16 \text{ mm} = -0.016 \text{ m}, \qquad z'_E = 10 \text{ mm} = 0.010 \text{ m}$$

$$\sigma_E = -\frac{M_{z'} y'_E}{I_{z'}} + \frac{M_{y'} z'_E}{I_{y'}} = -\frac{(112.763)(-0.016)}{53.6 \times 10^{-9}} + \frac{(41.042)(0.010)}{14.77 \times 10^{-9}}$$

$$= 61.448 \times 10^6 \text{ Pa} \qquad \sigma_E = 61.4 \text{ MPa}$$ ◄

Problem 4.137

***4.137 through *4.139** The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine the stress at point *A*.



$$I_y = 2\left\{\frac{1}{12}(180)(60)^3\right\} = 25.92 \times 10^6 \ mm^4$$

$$I_z = 2\left\{\frac{1}{12}(60)(180)^3 + (60)(180)(30)^2\right\} = 77.76 \times 10^6 \ mm^4$$

$$I_{yz} = 2\{(60)(180)(30)(30)\} = 19.44 \times 10^6 \ mm^4$$

Using Mohr's circle determine the principal axes and principal moments of inertia.

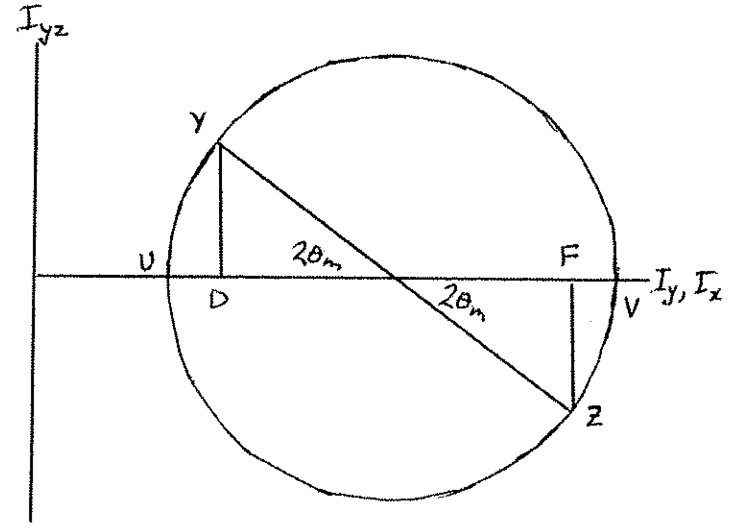
$Y = (25.92 \times 10^6 \ mm^4, \ 19.44 \times 10^6 \ mm^4)$

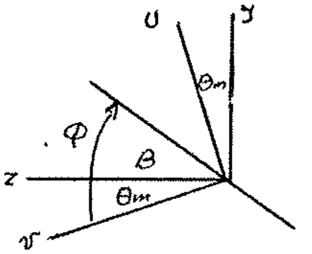
$Z = (77.76 \times 10^6 \ mm^4, \ -19.44 \times 10^6 \ mm^4)$

$E = (51.84 \times 10^6 \ mm^4, \ 0)$

$$\tan 2\theta_m = \frac{DY}{DE} = \frac{19.44}{25.92}$$

$2\theta_m = 36.87^\circ \qquad \theta_m = 18.435^\circ$





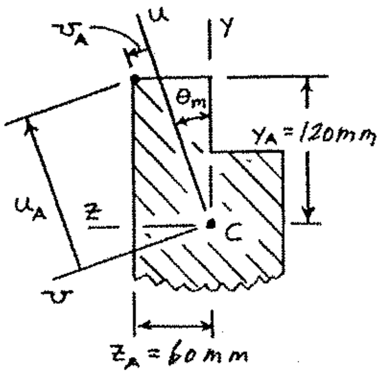
$R = \sqrt{\overline{DE}^2 + \overline{DY}^2} = 32.4 \times 10^6 \ mm^6$

$I_u = (51.84 - 32.4)10^6 = 19.44 \times 10^6 \ mm^4$

$I_v = (51.84 + 32.4)10^6$
$= 84.24 \times 10^6 \ mm^4$

$M_u = 14 \sin 18.435^\circ = 4.43 \ kNm$

$M_v = 14 \cos 18.435^\circ = 13.3 \ kNm$



$u_A = 0.12 \cos 18.435^\circ + 0.06 \sin 18.435^\circ = 0.1328 \ m$

$v_A = -0.12 \sin 18.435^\circ + 0.06 \cos 18.435^\circ = 0.01897 \ m$

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u}$$

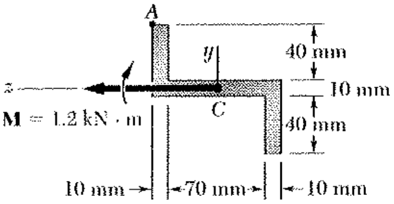
$$= -\frac{(13300)(0.1328)}{84.24 \times 10^{-6}} + \frac{(4430)(0.01897)}{19.44 \times 10^{-6}}$$

$$= -16.64 \ MPa$$

$\sigma_A = -16.6 \ MPa$ ◄

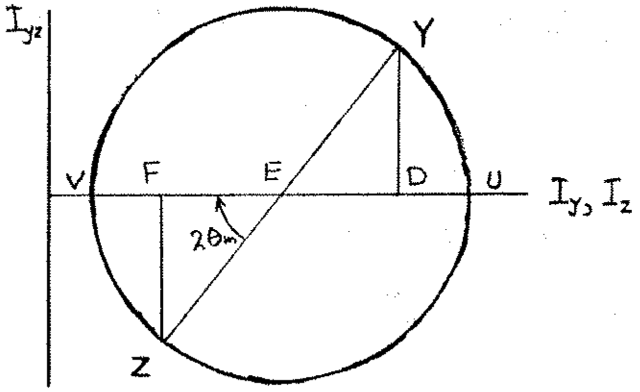
Problem 4.138

***4.137 through *4.139** The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine the stress at point *A*.



$I_y = 1.894 \times 10^6$ mm^4
$I_z = 0.614 \times 10^6$ mm^4
$I_{yz} = +0.800 \times 10^6$ mm^4

Using Mohr's circle, determine the principal axes and the principal moments of inertia.



$Y\,(1.894, 0.800) \times 10^6$ mm^4
$Z\,(0.614, 0.800) \times 10^6$ mm^4
$E\,(1.254, 0\;) \times 10^6$ mm^4

$$R = \sqrt{\overline{EF}^2 + \overline{FZ}^2} = \sqrt{0.640^2 + 0.800^2} \times 10^{-6} = 1.0245 \times 10^6 \text{ mm}^4$$

$$I_v = (1.254 - 1.0245) \times 10^6 \text{ mm}^4 = 0.2295 \times 10^6 \text{ mm}^4 = 0.2295 \times 10^{-6} \text{ m}^4$$

$$I_u = (1.254 + 1.0245) \times 10^6 \text{ mm}^4 = 2.2785 \times 10^6 \text{ mm}^4 = 2.2785 \times 10^{-6} \text{ m}^4$$

$$\tan 2\theta_n = \frac{FZ}{FE} = \frac{0.800 \times 10^6}{0.640 \times 10^6} = 1.25 \qquad \theta_m = 25.67°$$

$$M_v = M \cos \theta_m = (1.2 \times 10^3) \cos 25.67° = 1.0816 \times 10^3 \text{ N·m}$$

$$M_u = -M \sin \theta_m = -(1.2 \times 10^3) \sin 25.67° = -0.5198 \times 10^3 \text{ N·m}$$

$$u_A = y_A \cos \theta_m - z_A \sin \theta_m = 45 \cos 25.67° - 45 \sin 25.67° = 21.07 \text{ mm}$$

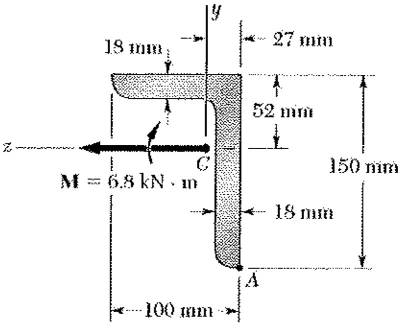
$$v_A = z_A \cos \theta_m + y_A \sin \theta_m = 45 \cos 25.67° + 45 \sin 25.67° = 60.05 \text{ mm}$$

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u} = -\frac{(1.0816 \times 10^3)(21.07 \times 10^{-3})}{0.2295 \times 10^{-6}} + \frac{(-0.5198 \times 10^3)(60.05 \times 10^{-3})}{2.2785 \times 10^{-6}}$$

$$= 113.0 \times 10^6 \text{ Pa} \qquad \sigma_A = 113.0 \text{ MPa} \blacktriangleleft$$

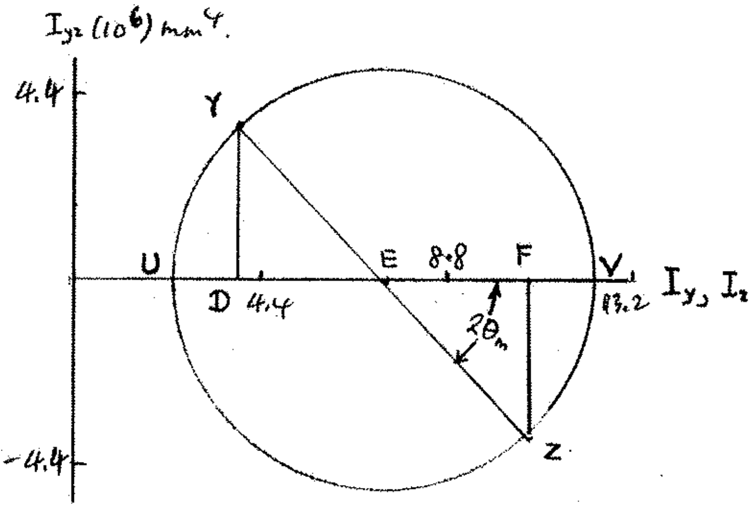
Problem 4.139

***4.137** through ***4.139** The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine the stress at point A.



$I_y = 3.65 \times 10^6$ mm^4
$I_z = 10.1 \times 10^6$ mm^4
$I_{yz} = 3.45 \times 10^6$ mm^4

Using Mohr's circle, determine the principal axes and principal moments of inertia



Y $(3.65, 3.45)10^6$ mm^4
Z $(10.1, 3.45)10^6$ mm^4
E $(6.875, 0)\ 10^6$ mm^4

$EF = 3.225 \times 10^6$ mm^4
$FZ = 3.45 \times 10^6$ mm^4

$R = \sqrt{3.225^2 + 3.45^2} = 4.72 \times 10^6$ mm^4 $\quad \tan 2\theta_m = \frac{FZ}{EF} = \frac{3.45}{3.225} = 1.0698$

$\theta_m = 23.5°$ $\quad I_u = (6.875 - 4.72)10^6 = 2.155 \times 10^6$ mm^4, $\quad I_v = (6.875 + 4.72)10^6 = 11.6 \times 10^6$ mm^4

$M_u = \quad = (6.8) \sin 23.5° = 2.71$ kNm

$M_v = \quad = (6.8) \cos 23.5° = 6.24$ kNm

$U_A = y_A \cos\theta_m + Z_A \sin\theta_m = -98 \cos 23.5° - 27 \sin 23.5 = -100.6$ mm

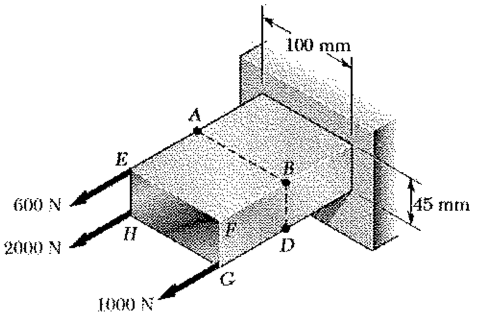
$V_A : Z_A \cos\theta_m - y_A \sin\theta_m = -27 \cos 23.5° + 98 \sin 23.5° = 14.3$ mm

$$\sigma_A = -\frac{M_v U_A}{I_v} + \frac{M_u V_A}{I_u} = -\frac{(6240)(-0.1006)}{11.6 \times 10^{-6}} + \frac{(2710)(0.0143)}{2.155 \times 10^{-6}}$$

$= 72.1$ MPa ◄

Problem 4.140

4.140 For the loading shown, determine (*a*) the stress at points *A* and *B*, (*b*) the point where the neutral axis intersects line *ABD*.

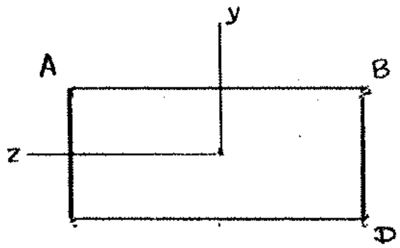


Add y and z axes as shown

$A = (100)(45) = 4500 \text{ mm}^2.$

$I_z = \frac{1}{12}(100)(45)^3 = 759375 \text{ mm}^4$

$I_y = \frac{1}{12}(45)(100)^3 = 3.75 \times 10^6 \text{ mm}^4$



Resultant force and bending couples

$P = 600 + 2000 + 1000 = 3600 \text{ N}$

$M_z = [-(600)(0.225) + (2000)(0.225) + (1000)(0.225)](1 \times 10^{-1})$
$= 54 \text{ N m m}$

$M_y = (600)(0.05) + (2000)(0.05) - (1000)(0.05)$
$= 80 \text{ N/m}$

(a) $\sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{3600}{4500 \times 10^{-6}} - \frac{(54)(0.0225)}{759375 \times 10^{-12}} + \frac{(80)(0.05)}{3.75 \times 10^{-6}} = 266.7 \text{ kPa}$ ◀

$\sigma_B = \frac{P}{A} - \frac{M_z' y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{3600}{4500 \times 10^{-6}} - \frac{(54)(0.0225)}{1.944} + \frac{(800)(-2)}{9.6} = -1.87 \text{ MPa}$ ◀

(b) Intersection of neutral axis with line AB or its extension.

$\sigma = 0, \quad y = 0.0225 \text{ m}, \; z = ?$

$0 = \frac{P}{A} - \frac{M_z Y}{I_z} + \frac{M_y Y}{I_y} = \frac{3600}{4500 \times 10^{-6}} - \frac{(54)(0.0225)}{759375 \times 10^{-12}} + \frac{80 z}{3.75 \times 10^{-6}}$

$-0.8 \times 10^6 + 21.33 z = 0 \qquad z = 0.0375 \text{ m}$

Intersects AB at 12.5 mm from A ◀

Intersection of neutral axis with line BD or its extension.

$\sigma = 0 \qquad z = -50 \text{ mm} \quad y = ?$

$0 = \frac{P}{A} - \frac{M_z Y}{I_z} + \frac{M_y z}{I_y} = \frac{3600}{4500 \times 10^{-6}} - \frac{54 y}{759375 \times 10^{-12}} + \frac{(80)(-0.05)}{3.75 \times 10^{-6}}$

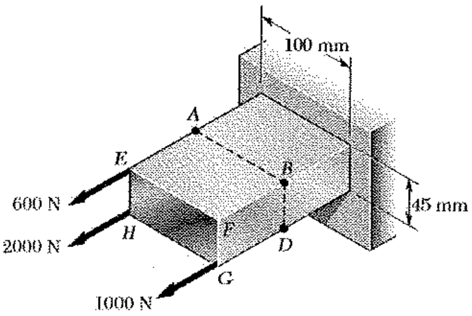
$-0.2667 \times 10^6 - 71.1111 y = 0 \qquad y = -0.00375 \text{ m}$

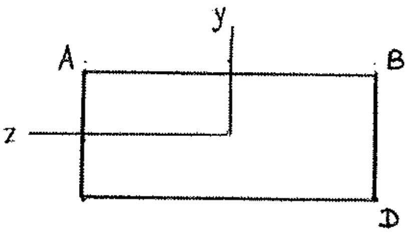
Intersects BD at 18.75 mm. from D ◀

Problem 4.141

4.141 Solve Prob. 4.140, assuming that the magnitude of the force applied at G is increased from 1.0 kN to 1.6 kN.

4.140 For the loading shown, determine (a) the stress at points A and B, (b) the point where the neutral axis intersects line ABD.





Add y and z axes as shown.

$A = (100 \times 45) = 4500 \text{ mm}^2$

$I_z = \frac{1}{12}(100)(45)^3 = 759375 \text{ mm}^4$

$I_y = \frac{1}{12}(45)(100)^3 = 3.75 \times 10^6 \text{ mm}^4$

Resultant force and bending couples

$P = 600 + 2000 + 1600 = 4200 \text{ N}.$

$M_z = -(600)(0.0225) + (2000)(0.0225) + (1600)(0.0225)$
$= 67.5 \text{ Nm}$

$M_y = (600)(0.05) + (2000)(0.05) - (1600)(0.05)$
$= 50 \text{ Nm}$

(a) $\sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{4200}{0.0045} - \frac{(67.5)(0.0225)}{759375 \times 10^{-12}} + \frac{(50)(0.05)}{3.75 \times 10^{-6}} = -400 \text{ kPa}$

$\sigma_B = \frac{P}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{4200}{0.0045} - \frac{(67.5)(0.0225)}{759375 \times 10^{-12}} + \frac{(50)(-0.05)}{3.75 \times 10^{-6}} = -1.73 \text{ MPa}$

(b) Intersection of neutral axis with line AB or its extension.

$\sigma = 0, \quad y = 0.9 \text{ in.} \quad z = ?$

$0 = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y} = \frac{4200}{0.0045} - \frac{(67.5)(0.0225)}{759375 \times 10^{-12}} + \frac{50z}{3.75 \times 10^{-6}}$

$-1.0667 \times 10^6 + 13.3333 z = 0 \qquad z = 0.08 \text{ m} = 80 \text{ mm}$

Does not intersect AB

Intersection of neutral axis with line BD or its extension.

$\sigma = 0 \quad z = -2 \text{ in.} \quad y = ?$

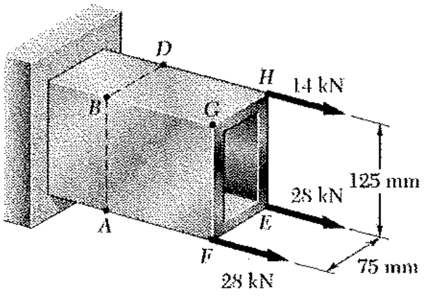
$0 = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y} = \frac{4200}{0.0045} - \frac{67.5y}{759375 \times 10^{-12}} + \frac{(50)(-0.05)}{3.75 \times 10^{-6}}$

$10^6 \times 0.26667 - 88.8889 y = 0 \qquad y = 0.003 \text{ m} = 3 \text{ mm}.$

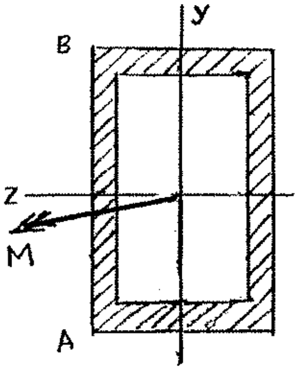
Intersects BD at 19.5 mm from B

Problem 4.142

4.142 The tube shown has a uniform wall thickness of 12 mm. For the loading given, determine (*a*) the stress at points *A* and *B*, (*b*) the point where the neutral axis intersects line *ABD*.



Add y- and z-axes as shown. Cross section is a 75 mm × 125 mm rectangle with a 51 mm × 101 mm rectangular cutout.



$$I_z = \frac{1}{12}(75)(125)^3 - \frac{1}{12}(51)(101)^3 = 7.8283\times10^6\ \text{mm}^4$$
$$= 7.8283\times10^{-6}\ \text{m}^4$$

$$I_y = \frac{1}{12}(125)(75)^3 - \frac{1}{12}(101)(51)^3 = 3.2781\times10^3\ \text{mm}^4$$
$$= 3.2781\times10^{-6}\ \text{m}^4$$

$$A = (75)(125) - (51)(101) = 4.224\times10^3\ \text{mm}^2$$
$$= 4.224\times10^{-3}\ \text{m}^2$$

Resultant force and bending couples:

$$P = 14 + 28 + 28 = 70\ \text{kN} = 70\times10^3\ \text{N}$$

$$M_z = -(62.5\,\text{mm})(14\,\text{kN}) + (62.5\,\text{mm})(28\,\text{kN}) + (62.5\,\text{mm})(28\,\text{kN})$$
$$= 2625\ \text{N}\cdot\text{m}$$

$$M_y = -(37.5\,\text{mm})(14\,\text{kN}) + (37.5\,\text{mm})(28\,\text{kN}) + (37.5\,\text{mm})(28\,\text{kN})$$
$$= -525\ \text{N}\cdot\text{m}$$

(a) $$\sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{70\times10^3}{4.224\times10^{-3}} - \frac{(2625)(-0.0625)}{7.8283\times10^{-6}} + \frac{(-525)(0.0375)}{3.2781\times10^{-6}}$$
$$= 31.524\times10^6\ \text{Pa}$$

$\sigma_A = 31.5\ \text{MPa}$ ◄

$$\sigma_B = \frac{P}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{70\times10^3}{4.224\times10^{-3}} - \frac{(2625)(0.0625)}{7.8283\times10^{-6}} + \frac{(-525)(0.0375)}{3.2781\times10^{-6}}$$
$$= -10.39\times10^6\ \text{Pa}$$

$\sigma_B = -10.39\ \text{MPa}$ ◄

(b) Let point H be the point where the neutral axis intersects AB.

$z_H = 0.0375\ \text{m}, \quad y_H = ?, \quad \sigma_H = 0$

$$0 = \frac{P}{A} - \frac{M_z y_H}{I_z} + \frac{M_y z_H}{I_y}$$

$$y_H = \frac{I_z}{M_z}\left(\frac{P}{A} + \frac{M z_H}{I_y}\right) = \frac{7.8283\times10^{-6}}{2625}\left[\frac{70\times10^3}{4.224\times10^{-3}} + \frac{(-525)(0.0375)}{3.2781\times10^{-6}}\right]$$
$$= 0.03151\ \text{m} = 31.51\ \text{mm}$$

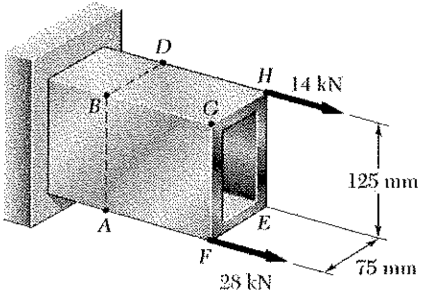
$31.51 + 62.5 = 94.0\ \text{mm}$

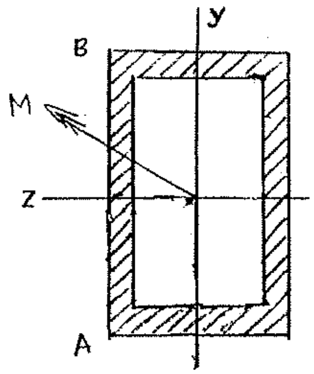
Answer: 94.0 mm above point A. ◄

Problem 4.143

4.143 Solve Prob. 4.142, assuming that the 28-kN force at point E is removed.

4.142 The tube shown has a uniform wall thickness of 12 mm. For the loading given, determine (*a*) the stress at points A and B, (*b*) the point where the neutral axis intersects line ABD.





Add y- and z-axes as shown. Cross section is a 75 mm × 125 mm rectangle with a 51 mm × 101 mm rectangular cutout.

$$I_z = \tfrac{1}{12}(75)(125)^3 - \tfrac{1}{12}(51)(101)^3 = 7.8283\times10^6\ \text{mm}^4$$
$$= 7.8283\times10^{-6}\ \text{m}^6$$
$$I_y = \tfrac{1}{12}(125)(75)^3 - \tfrac{1}{12}(101)(51)^3 = 3.2781\times10^6\ \text{mm}^4$$
$$= 3.2781\times10^{-6}\ \text{m}^4$$
$$A = (75)(125) - (51)(101) = 4.224\times10^3\ \text{mm}^2$$
$$= 4.224\times10^{-3}\ \text{m}^2$$

Resultant force and bending couples:

$$P = 14 + 28 = 42\ \text{kN} = 42\times10^3\ \text{N}$$
$$M_z = -(62.5\text{mm})(14\ \text{kN}) + (62.5\text{mm})(28\ \text{kN})$$
$$= 875\ \text{N}\cdot\text{m}$$
$$M_y = -(37.5\text{mm})(14\ \text{kN}) - (37.5\ \text{mm})(28\ \text{kN})$$
$$= 525\ \text{N}\cdot\text{m}$$

(a) $$\sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{42\times10^3}{4.224\times10^{-3}} - \frac{(875)(-0.0625)}{7.8283\times10^{-6}} + \frac{(525)(0.0375)}{3.2781\times10^{-6}}$$
$$= 22.935\times10^6\ \text{Pa} \qquad \sigma_A = 22.9\ \text{MPa}$$ ◄

$$\sigma_B = \frac{P}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{42\times10^3}{4.224\times10^{-3}} - \frac{(875)(0.0625)}{7.8283\times10^{-6}} + \frac{(525)(0.0375)}{3.2781\times10^{-6}}$$
$$= 8.9631\times10^6\ \text{Pa} \qquad \sigma_B = 8.96\ \text{MPa}$$ ◄

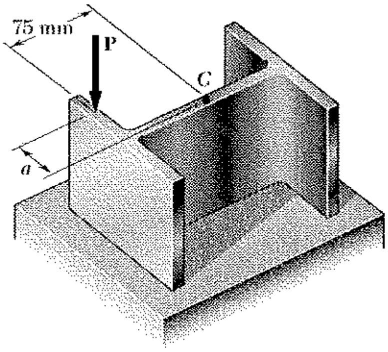
(b) Let point K be the point where the neutral axis intersects BD.

$$z_K = ?,\quad y_K = 0\ 0625\ \text{m},\quad \sigma_H = 0$$
$$0 = \frac{P}{A} - \frac{M_z y_H}{I_z} + \frac{M_y z_H}{I_y}$$
$$z_H = \frac{I_y}{M_y}\left(\frac{M_z y_H}{I_z} - \frac{P}{A}\right) = \frac{3.2781\times10^{-6}}{525}\left[\frac{(875)(0.0625)}{7.8283\times10^{-6}} - \frac{42\times10^3}{4.224\times10^{-3}}\right]$$
$$= -0.018465\ \text{m} = -18.465\ \text{mm}$$
$$37.5 + 18.465 = 56.0\ \text{mm}$$

Answer: 56.0 mm to the right of point B. ◄

Problem 4.144

4.144 An axial load **P** of magnitude 50 kN is applied as shown to a short section of a W150 × 24 rolled-steel member. Determine the largest distance *a* for which the maximum compressive stress does not exceed 90 MPa.



Add y- and z- axes.

For W 150 × 24 rolled-steel section

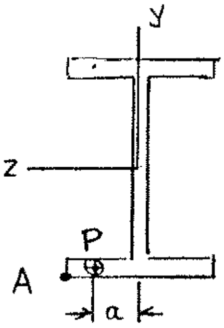
$A = 3060 \text{ mm}^2 = 3060 \times 10^{-6} \text{ m}^2$

$I_z = 13.4 \times 10^6 \text{ mm}^4 = 13.4 \times 10^{-6} \text{ m}^4$

$I_y = 1.83 \times 10^6 \text{ mm}^4 = 1.83 \times 10^{-6} \text{ m}^4$

$d = 160 \text{ mm}, \quad b_f = 102 \text{ mm}$

$y_A = -\frac{d}{2} = -80 \text{ mm}, \quad z_A = \frac{b_f}{2} = 51 \text{ mm}.$



$P = 50 \times 10^3 \text{ N}$

$M_z = -(50 \times 10^3)(75 \times 10^{-3}) = -3.75 \times 10^3 \text{ N} \cdot \text{m}$

$M_y = -Pa$

$$\sigma_A = -\frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} \qquad \sigma_A = -90 \times 10^6 \text{ Pa}$$

$$M_y = \frac{I_y}{z_A}\left\{\frac{M_z y_A}{I_z} + \frac{P}{A} + \sigma_A\right\}$$

$$= \frac{1.83 \times 10^{-6}}{51 \times 10^{-3}}\left\{\frac{(-3.75 \times 10^3)(-80 \times 10^{-3})}{13.4 \times 10^{-6}} + \frac{50 \times 10^3}{3060 \times 10^{-6}} + (-90 \times 10^6)\right\}$$

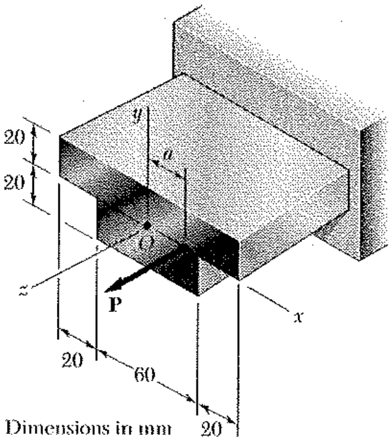
$$= \frac{1.83 \times 10^{-6}}{51 \times 10^{-3}}\left\{+22.388 + 16.340 - 90\right\} \times 10^6$$

$$= -1.8398 \times 10^3 \text{ N} \cdot \text{m}$$

$$a = -\frac{M_y}{P} = -\frac{-1.8398 \times 10^3}{50 \times 10^3} = 36.8 \times 10^{-3} \text{ m} \qquad a = 36.8 \text{ mm} \blacktriangleleft$$

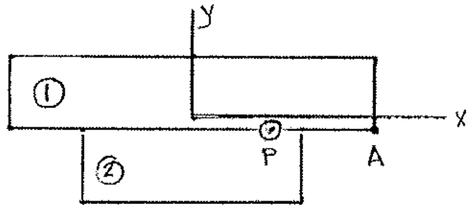
Problem 4.145

4.145 A horizontal load **P** of magnitude 100 kN is applied to the beam shown. Determine the largest distance *a* for which the maximum tensile stress in the beam does not exceed 75 MPa.



Dimensions in mm

Locate the centroid.



	A, mm^2	$\bar{y}$, mm	$A\bar{y}$, mm^3
①	2000	10	20×10^3
②	1200	−10	-12×10^3
Σ	3200		8×10^3

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{8 \times 10^3}{3200} = 2.5 \text{ mm}$$

Move coordinate origin to the centroid.

Coordinates of load point: $x_P = a$, $y_P = -2.5$ mm

Bending couples $M_x = y_P P$ $M_y = -aP$

$$I_x = \tfrac{1}{12}(100)(20)^3 + (2000)(7.5)^2 + \tfrac{1}{12}(60)(20)^3 + (1200)(12.5)^2 = 0.40667 \times 10^6 \text{ mm}^4$$
$$= 0.40667 \times 10^{-6} \text{ m}^4$$

$$I_y = \tfrac{1}{12}(20)(100)^3 + \tfrac{1}{12}(20)(60)^3 = 2.0267 \times 10^6 \text{ mm}^4 = 2.0267 \times 10^{-6} \text{ m}^4$$

$$\sigma = \frac{P}{A} + \frac{M_x y}{I_x} - \frac{M_y x}{I_y}$$

$\sigma_A = 75 \times 10^6$ Pa, $P = 100 \times 10^3$ N

$$M_y = \frac{I_y}{x}\left\{\frac{P}{A} + \frac{M_x y}{I_y} - \sigma\right\}$$

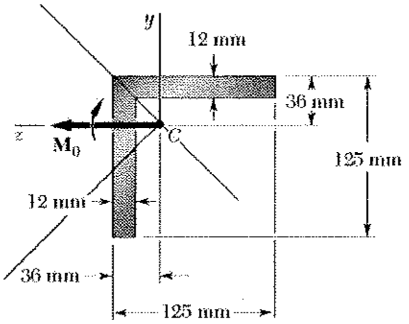
For point A $x = 50$ mm, $y = -2.5$ mm

$$M_y = \frac{2.0267 \times 10^{-6}}{50 \times 10^{-3}}\left\{\frac{100 \times 10^3}{3200 \times 10^{-6}} + \frac{(-2.5)(100 \times 10^3)(-2.5 \times 10^{-3})}{0.40667 \times 10^{-6}} - 75 \times 10^6\right\}$$
$$= \frac{2.0267 \times 10^{-6}}{50 \times 10^{-3}}\left\{31.25 + 1.537 - 75\right\} \times 10^6 = -1.7111 \times 10^3 \text{ N·m}$$

$$a = -\frac{M_y}{P} = -\frac{(1.7111 \times 10^3)}{100 \times 10^3} = 17.11 \times 10^3 \text{ m}$$

$a = 17.11$ mm ◄

Problem 4.146



4.146 A beam having the cross section shown is subjected to a couple $\mathbf{M}_0$ that acts in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress in the beam is not to exceed 84 MPa. *Given:* $I_y = I_z = 4.7 \times 10^6\ \text{mm}^4$, $A = 3064\ \text{mm}^2$, $k_{\min} = 25$ mm. (*Hint:* By reason of symmetry, the principal axes form an angle of 45° with the coordinate axes. Use the relations $I_{\min} = Ak^2_{\min}$ and $I_{\min} + I_{\max} = I_y + I_z$.)

$$M_u = M_o \sin 45^\circ = 0.70711\ M_o$$

$$M_v = M_o \cos 45^\circ = 0.7071\ M_o$$

$$I_{min} = A k_{min}^2 = (3064)(25)^2 = 1.915 \times 10^6\ mm^4$$

$$I_{max} = I_y + I_z - I_{min} = (4.7 + 4.7 - 1.915)10^6 = 7.485 \times 10^6\ mm^4$$

$$u_B = y_B \cos 45^\circ + z_B \sin 45^\circ = -89 \cos 45^\circ + 24 \sin 45^\circ = -46\ mm$$

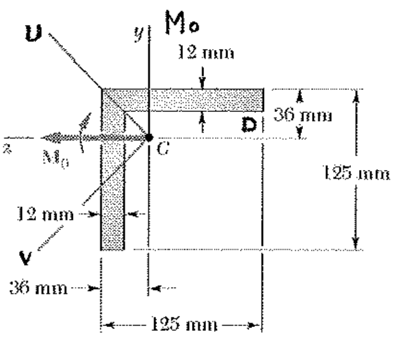
$$v_B = z_B \cos 45^\circ - y_B \sin 45^\circ = 24 \cos 45^\circ - (-89) \sin 45^\circ = 80\ mm.$$

$$\sigma_B = -\frac{M_v u_B}{I_v} + \frac{M_u v_B}{I_u} = 0.70711\ M_o \left[-\frac{u_B}{I_{min}} + \frac{v_B}{I_{max}} \right]$$

$$= 0.70711\ M_o \left[-\frac{(-0.046)}{1.915 \times 10^{-6}} + \frac{0.08}{7.485 \times 10^{-6}} \right] = 24543\ M_o$$

$$M_o = \frac{\sigma_B}{24543} = \frac{84 \times 10^6}{24543} = 3.42\ kN\ m.$$ ◄

Problem 4.147



4.147 Solve Prob. 4.146, assuming that the couple $\mathbf{M}_0$ acts in a horizontal plane.

4.146 A beam having the cross section shown is subjected to a couple $\mathbf{M}_0$ that acts in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress in the beam is not to exceed 84 MPa. *Given*: $I_y = I_z = 4.7 \times 10^6$ mm^4, $A = 3064$ mm^2, $k_{min} = 25$ mm. (*Hint*: By reason of symmetry, the principal axes form an angle of 45° with the coordinate axes. Use the relations $I_{min} = Ak^2_{min}$ and $I_{min} + I_{max} = I_y + I_z$.)

$$M_U = M_o \cos 45^\circ = 0.70711 M_o$$

$$M_V = -M_o \sin 45^\circ = -0.70711 M_o$$

$$I_{min} = A k_{min}^2 = (3064)(25)^2 = 1.915 \times 10^6 \text{ mm}^4$$

$$I_{max} = I_y + I_z - I_{min} = 10^6(4.7 + 4.7 + 1.915) = 7.485 \times 10^6 \text{ mm}^4$$

$$U_D = y_0 \cos 45^\circ + z_D \sin 45^\circ = 24 \cos 45^\circ + (-89 \sin 45^\circ) = -46\text{mm}$$

$$V_D = z_D \cos 45^\circ - y_0 \sin 45^\circ = (-89)\cos 45^\circ - (24) \sin 45^\circ = 80\text{mm}$$

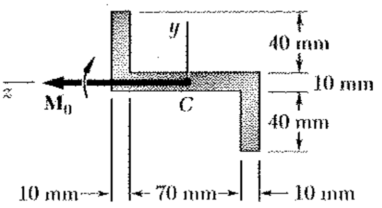
$$\sigma_D = -\frac{M_V U_D}{I_V} + \frac{M_U V_D}{I_U} = 0.70711 M_o \left[-\frac{U_D}{I_{min}} + \frac{V_D}{I_{max}} \right]$$

$$= 0.70711 M_o \left[-\frac{(-0.046)}{1.915 \times 10^6} + \frac{0.08}{7.485 \times 10^6} \right] = 24543 M_o$$

$$M_o = \frac{\sigma_D}{24543} = \frac{12}{24543} = 3.42 \text{ kN m}.$$

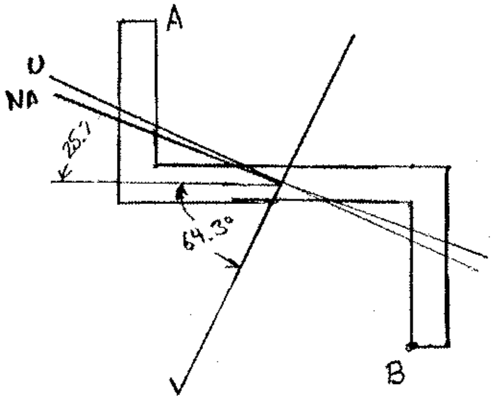
Problem 4.148

4.148 The Z section shown is subjected to a couple $\mathbf{M}_0$ acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 80 MPa. *Given*: $I_{max} = 2.28 \times 10^{-6}\ \text{mm}^4$, $I_{min} = 0.23 \times 10^{-6}\ \text{mm}^4$, principal axes 25.7° ⦩ and 64.3° ∠.



$$I_v = I_{max} = 2.28\times10^{6}\ \text{mm}^4 = 2.28\times10^{-6}\ \text{m}^4$$

$$I_u = I_{min} = 0.23\times10^{6}\ \text{mm}^4 = 0.23\times10^{-6}\ \text{m}^4$$



$$M_v = M_o \cos 64.3°$$

$$M_u = M_o \sin 64.3°$$

$$\theta = 64.3°$$

$$\tan\varphi = \frac{I_v}{I_u}\tan\theta$$

$$= \frac{2.28\times10^{-6}}{0.23\times10^{-6}}\tan 64.3° = 20.597$$

$$\varphi = 87.22°$$

Points A and B are farthest from the neutral axis.

$$u_B = y_B\cos 64.3° + z_B \sin 64.3° = (-45)\cos 64.3° + (-35)\sin 64.3°$$

$$= -51.05\ \text{mm}$$

$$v_B = z_B\cos 64.3° - y_B\sin 64.3° = (-35)\cos 64.3° - (-45)\sin 64.3°$$

$$= +25.37\ \text{mm}$$

$$\sigma_B = -\frac{M_v u_B}{I_v} + \frac{M_u v_B}{I_u}$$

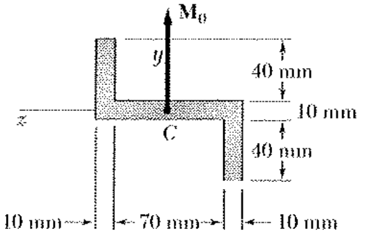
$$80\times10^6 = -\frac{(M_o\cos 64.3°)(-51.05\times10^{-3})}{2.28\times10^{-6}} + \frac{(M_o \sin 64.3°)(25.37\times10^{-3})}{0.23\times10^{-6}}$$

$$= 93.81\times10^3\ M_o$$

$$M_o = \frac{80\times10^6}{109.1\times10^3}$$

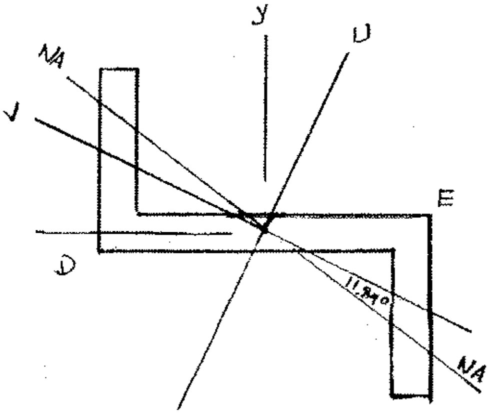
$M_o = 733\ \text{N}\cdot\text{m}$ ◄

Problem 4.149



4.149 Solve Prob. 4.148 assuming that the couple $\mathbf{M}_0$ acts in a horizontal plane.

4.148 The Z section shown is subjected to a couple $\mathbf{M}_0$ acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 80 MPa. *Given*: $I_{max} = 2.28 \times 10^{-6}\ \text{mm}^4$, $I_{min} = 0.23 \times 10^{-6}\ \text{mm}^4$, principal axes 25.7° ⦣ and 64.3° ⦣.



$I_v = I_{min} = 0.23 \times 10^6\ mm^4 = 0.23 \times 10^6\ m^4$

$I_u = I_{max} = 2.23 \times 10^6\ mm^4 = 2.23 \times 10^6\ m^4$

$M_v = M_o \cos 64.3°$

$M_u = M_o \sin 64.3°$

$\theta = 64.3°$

$$\tan\varphi = \frac{I_v}{I_u}\tan\theta$$

$$= \frac{0.23\times10^{-6}}{2.28\times10^{-6}}\tan 64.3° = 0.20961$$

$\varphi = 11.84°$

Points D and E are farthest from the neutral axis.

$u_D = y_D \cos 25.7° - z_D \sin 25.7° = (-5)\cos 25.7° - 45 \sin 25.7°$

$= -24.02\ mm$

$v_D = z_D \cos 25.7° + y_D \sin 25.7° = 45 \cos 25.7° + (-5) \sin 25.7°$

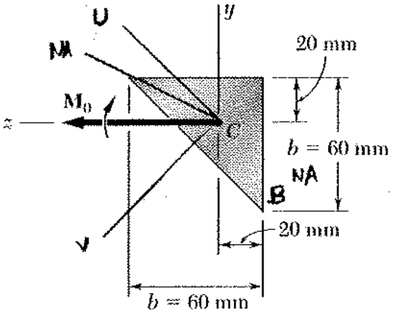
$= 38.38\ mm$

$$\sigma_D = -\frac{M_v u_D}{I_v} + \frac{M_u v_D}{I_u} = -\frac{(M_o \cos 64.3°)(-24.02\times10^{-3})}{0.23\times10^{-6}} + \frac{(M_o \sin 64.3°)(38.38\times10^{-3})}{2.28\times10^{-6}}$$

$80\times10^6 = 60.48\times10^3\ M_o$

$M_o = 1.323 \times 10^3\ N\cdot m$ $\qquad$ $M_o = 1.323\ kN\cdot m$ ◄

Problem 4.150



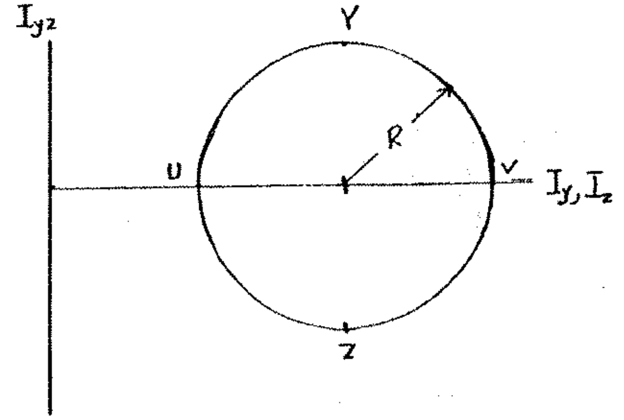
4.150 A beam having the cross section shown is subjected to a couple $\mathbf{M}_0$ acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 100 MPa. *Given*: $I_y = I_z = b^4/36$ and $I_{yz} = b^4/72$.

$$I_y = I_z = \frac{b^4}{36} = \frac{60^4}{36} = 0.360 \times 10^6 \text{ mm}^4$$

$$I_{yz} = \frac{b^4}{72} = \frac{60^4}{72} = 0.180 \times 10^6 \text{ mm}^4$$

Principal axes are symmetry axes.

Using Mohr's circle determine the principal moments of inertia.



$$R = |I_{yz}| = 0.180 \times 10^6 \text{ mm}^4$$

$$I_V = \frac{I_y + I_z}{2} + R$$

$$= 0.540 \times 10^6 \text{ mm}^4 = 0.540 \times 10^{-6} \text{ m}^4$$

$$I_U = \frac{I_y + I_z}{2} - R$$

$$= 0.180 \times 10^6 \text{ mm}^4 = 0.180 \times 10^{-6} \text{ m}^4$$

$M_U = M_o \sin 45° = 0.70711\, M_o$, $\quad M_V = M_o \cos 45° = 0.70711\, M_o$

$\theta = 45°$ $\qquad \tan\varphi = \frac{I_V}{I_U}\tan\theta = \frac{0.540 \times 10^{-6}}{0.180 \times 10^{-6}} \tan 45° = 3$

$$\varphi = 71.56°$$

Point A: $U_A = 0.$ $\quad V_A = -20\sqrt{2}$ mm

$$\sigma_A = -\frac{M_V U_A}{I_V} + \frac{M_U V_A}{I_U} = 0 + \frac{(0.70711\, M_o)(-20\sqrt{2} \times 10^{-3})}{0.180 \times 10^{-6}} = -11.11 \times 10^3\, M_o$$

$$M_o = -\frac{\sigma_A}{111.11 \times 10^3} = -\frac{-100 \times 10^6}{111.11 \times 10^3} = 900 \text{ N·m}$$

Point B: $U_B = -\frac{60}{\sqrt{2}}$ mm, $\quad V_B = \frac{20}{\sqrt{2}}$ mm

$$\sigma_B = -\frac{M_V U_B}{I_V} + \frac{M_U V_B}{I_U} = -\frac{(0.70711\, M_o)(-\frac{60}{\sqrt{2}} \times 10^{-3})}{0.540 \times 10^{-6}} + \frac{(0.70711\, M_o)(\frac{20}{\sqrt{2}} \times 10^{-3})}{0.180 \times 10^{-6}}$$

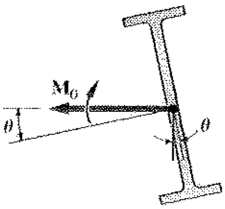
$$= 111.11 \times 10^3\, M_o$$

$$M_o = \frac{\sigma_B}{111.11 \times 10^{-3}} = \frac{100 \times 10^6}{111.11 \times 10^{-3}} = 900 \text{ N-m}$$

Choose the smaller value. $\qquad M_o = 900$ N·m ◀

Problem 4.151

4.151 A couple $\mathbf{M}_0$ acting in a vertical plane is applied to a W310 × 23.8 rolled-steel beam, whose web forms an angle θ with the vertical. Denoting by σ_0 the maximum stress in the beam when $\theta = 0$, determine the angle of inclination θ of the beam for which the maximum stress is $2\sigma_0$.



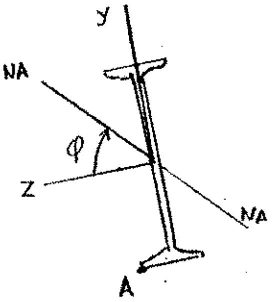
For W 310X23·6. rolled steel section

$I_z = 42{\cdot}7 \times 10^6\, mm^4$. $I_y = 1{\cdot}16 \times 10^6\, mm^4$

$d = 305\, mm$ $b_f = 101\, mm$

$y_A = -\frac{d}{2}$ $z_A = \frac{b_f}{2}$

$$\tan\varphi = \frac{I_z}{I_y}\tan\theta = \frac{42{\cdot}7}{1{\cdot}16}\tan\theta = 36{\cdot}81 \tan\theta$$



Point A is farthest from the neutral axis.

$M_y = M_0 \sin\theta$ $M_z = M_0 \cos\theta$

$$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{M_0 d}{2I_z}\cos\theta + \frac{M_0 b_f}{2I_y}\sin\theta$$

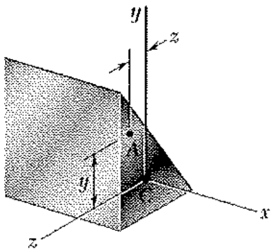
$$= \frac{M_0 d}{2I_z}\left(1 + \frac{I_z b_f}{I_y d}\tan\theta\right)$$

For $\theta = 0$ $\sigma_0 = \frac{M_0 d}{2I_z}$

$$\sigma_A = \sigma_0\left(1 + \frac{I_z b_f}{I_y d}\tan\theta\right) = 2\sigma_0$$

$$\tan\theta = \frac{I_y d}{I_z b_f} = \frac{(1{\cdot}16)(305)}{(42{\cdot}7)(101)} = 0{\cdot}08204 \qquad \theta = 4.70^\circ$$ ◄

Problem 4.152



4.152 A beam of unsymmetric cross section is subjected to a couple $\mathbf{M}_0$ acting in the vertical plane xy. Show that the stress at point A, of coordinates y and z, is

$$\sigma_A = -\frac{yI_y - zI_{yz}}{I_y I_z - I_{yz}^2} M_z$$

where I_y, I_z, and I_{yz} denote the moments and product of inertia of the cross section with respect to the coordinate axes, and M_z the moment of the couple.

The stress σ_A varies linearly with the coordinates y and z. Since the axial force is zero, the y- and z-axes are centroidal axes.

$\sigma_A = C_1 y + C_2 z$ where C_1 and C_2 are constants.

$$M_y = \int z\sigma_A\, dA = C_1 \int yz\, dA + C_2 \int z^2 dA$$

$$= I_{yz} C_1 + I_y C_2 = 0$$

$$C_2 = -\frac{I_{yz}}{I_y} C_1$$

$$M_z = -\int y \sigma_A\, dz = -C_1 \int y^2\, dA + C_2 \int yz\, dA$$

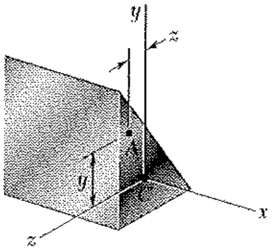
$$= -I_z C_1 - I_{yz} \frac{I_{yz}}{I_y} C_1$$

$$I_y M_z = -(I_y I_z - I_{yz}^2)\, C_1$$

$$C_1 = -\frac{I_y M_z}{I_y I_z - I_{yz}^2} \qquad C_2 = +\frac{I_{yz} M_z}{I_y I_z - I_{yz}^2}$$

$$\sigma_A = -\frac{I_y y - I_{yz} z}{I_y I_z - I_{yz}^2} M_z$$ ◄

Problem 4.153



4.153 A beam of unsymmetric cross section is subjected to a couple $\mathbf{M}_0$ acting in the horizontal plane xz. Show that the stress at point A, of coordinates y and z, is

$$\sigma_A = \frac{zI_z - yI_{yz}}{I_y I_z - I_{yz}^2} M_y$$

where I_y, I_z, and I_{yz} denote the moments and product of inertia of the cross section with respect to the coordinate axes, and M_y the moment of the couple.

The stress σ_A varies linearly with the coordinates y and z. Since the axial force is zero, the y- and z-axes are centroidal axes.

$\sigma_A = C_1 y + C_2 z$ where C_1 and C_2 are constants.

$$M_z = -\int y \sigma_A \, dA = -C_1 \int y^2 \, dA - C_2 \int yz \, dA$$

$$= -I_z C_1 - I_{yz} C_2 = 0$$

$$C_1 = -\frac{I_{yz}}{I_z} C_2$$

$$M_y = \int z \sigma_A \, dA = C_1 \int yz \, dA + C_2 \int z^2 \, dA$$

$$= I_{yz} C_1 + I_y C_2$$

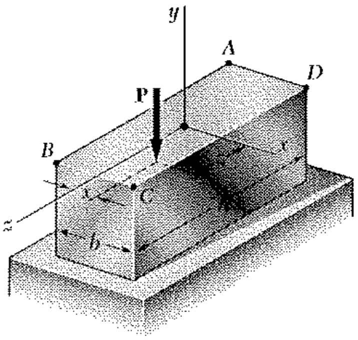
$$-I_{yz} \frac{I_{yz}}{I_z} C_2 + I_y C_2$$

$$I_z M_y = (I_y I_z - I_{yz}^2) C_2$$

$$C_2 = \frac{I_z M_y}{I_y I_z - I_{yz}^2} \qquad C_1 = -\frac{I_{yz} M_y}{I_y I_z - I_{yz}^2}$$

$$\sigma_A = \frac{I_z z - I_{yz} y}{I_y I_z - I_{yz}^2} M_y$$

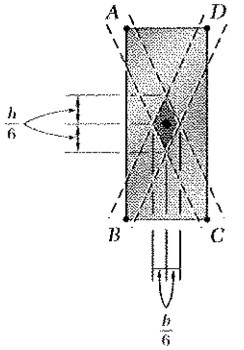
Problem 4.154



4.154 (*a*) Show that the stress at corner *A* of the prismatic member shown in Fig. P4.154*a* will be zero if the vertical force **P** is applied at a point located on the line

$$\frac{x}{b/6} + \frac{z}{h/6} = 1$$

(*b*) Further show that, if no tensile stress is to occur in the member, the force **P** must be applied at a point located within the area bounded by the line found in part *a* and three similar lines corresponding to the condition of zero stress at *B, C,* and *D,* respectively. This area, shown in Fig. 4.154*b*, is known as the *kern* of the cross section.



$I_z = \frac{1}{12} h b^3$ $\qquad$ $I_x = \frac{1}{12} b h^3$ $\qquad$ $A = bh$

$z_A = -\frac{h}{2}$ $\qquad$ $x_A = -\frac{b}{2}$

Let P be the load point.

$M_z = -P x_P$ $\qquad$ $M_x = P z_P$

$$\sigma_A = -\frac{P}{A} + \frac{M_z x_A}{I_z} - \frac{M_x z_A}{I_x}$$

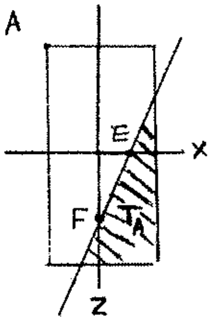
$$= -\frac{P}{bh} + \frac{(-P x_P)(-\frac{b}{2})}{\frac{1}{12} h b^3} - \frac{P z_P (-\frac{h}{2})}{\frac{1}{12} b h^3}$$

$$= -\frac{P}{bh}\left[1 - \frac{x_P}{b/6} - \frac{z_P}{h/6}\right]$$

For $\sigma_A = 0$, $\qquad 1 - \frac{x}{b/6} - \frac{z}{h/6} = 0, \qquad \frac{x}{b/6} + \frac{z}{h/6} = 1$

At point E: $z = 0$ ∴ $x_E = b/6$

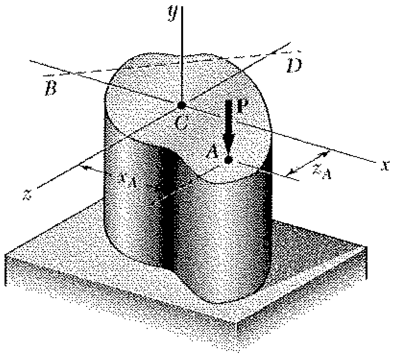
At point F: $x = 0$ ∴ $z_F = h/6$



If the line of action (x_P, z_P) lies within the portion marked T_A, a tensile will occur at corner A.

By considering $\sigma_B = 0$, $\sigma_C = 0$, and $\sigma_D = 0$, the other portions producing tensile stresses are identified.

Problem 4.155



4.155 (*a*) Show that, if a vertical force **P** is applied at point *A* of the section shown, the equation of the neutral axis *BD* is

$$\left(\frac{x_A}{k_z^2}\right)x + \left(\frac{z_A}{k_x^2}\right)z = -1$$

where k_z and k_x denote the radius of gyration of the cross section with respect to the *z* axis and the *x* axis, respectively. (*b*) Further show that, if a vertical force **Q** is applied at any point located on line *BD*, the stress at point *A* will be zero.

Definitions: $k_x^2 = \frac{I_x}{A}$, $k_z^2 = \frac{I_z}{A}$

(a) $M_x = Pz_A \qquad M_z = -Px_A$

$$\sigma_E = -\frac{P}{A} + \frac{M_z x_E}{I_z} - \frac{M_x z_E}{I_x} = -\frac{P}{A} - \frac{Px_A x_E}{Ak_z^2} - \frac{Pz_A z_E}{Ak_x^2}$$

$$= -\frac{P}{A}\left[1 + \left(\frac{x_A}{k_z^2}\right)x_E + \left(\frac{z_A}{k_x^2}\right)z_E\right] = 0 \text{ if } E \text{ lies on neutral axis.}$$

$$1 + \left(\frac{x_A}{k_z^2}\right)x + \left(\frac{z_A}{k_x^2}\right)z = 0, \qquad \left(\frac{x_A}{k_z^2}\right)x + \left(\frac{z_A}{k_x^2}\right)z = -1$$ ◄

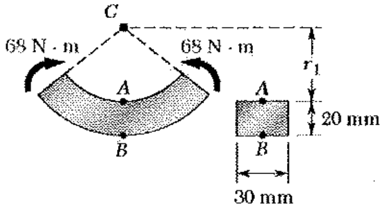
(b) $M_x = Pz_E \qquad M_z = -Px_E$

$$\sigma_A = -\frac{P}{A} + \frac{M_z x_A}{I_z} - \frac{M_x z_A}{I_y} = -\frac{P}{A} - \frac{Px_E x_A}{Ak_z^2} - \frac{Pz_E z_A}{Ak_x^2}$$

$= 0$ by equation from Part (a). ◄

Problem 4.156

4.156 For the curved bar and loading shown, determine the stress point A when (*a*) $r_1 = 30$ mm, (*b*) $r_1 = 50$ mm.



$h = 20\,mm \qquad A = (30)(20) = 600\ mm^2$

(a) $r_1 = 30mm \qquad r_2 = 50mm$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{20}{\ln \frac{5}{3}} = 39.15\ mm.$$

$$\bar{r} = \tfrac{1}{2}(r_1 + r_2) = 40mm.$$

$$e = \bar{r} - R = 0.85mm.$$

$y_A = 39.15 - 30 = 9.15\ mm \qquad r_A = r_1 = 30\ mm.$

$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(68)(0.00915)}{(600\times10^{-6})(0.00085)(0.03)} = -4.07\ MPa$$

$-4.07 MPa$ ◄

(b) $r_1 = 50\,mm. \qquad r_2 = 70\,mm$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{20}{\ln \frac{7}{5}} = 59.44\,mm.$$

$$\bar{r} = \tfrac{1}{2}(r_1 + r_2) = 60\ mm.$$

$$e = \bar{r} - R = 0.56\ mm.$$

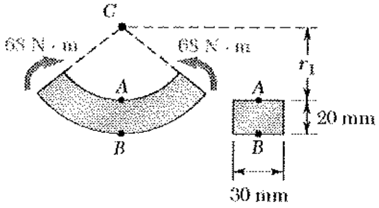
$y_A = 59.44 - 50 = 9.44\ mm \qquad r_A = r_1 = 50\,mm$

$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(68)(0.00944)}{(600\times10^{-6})(0.00056)(0.05)} = -38.2\ MPa$$

$-38.2\ MPa$ ◄

Problem 4.157

4.157 For the curved bar and loading shown, determine the stress points *A* and *B* when $r_1 = 40$ mm.



$h = 20 mm \quad r_1 = 40 mm \quad r_2 = 60 mm.$

$A = (30)(20) = 600 \; mm^2.$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{20}{\ln \frac{60}{40}} = 49.3261 \; mm$$

$\bar{r} = \frac{1}{2}(r_1 + r_2) = 50 mm.$

$e = \bar{r} - R = 0.6739 \; mm$

$y_A = 49.3261 - 40 = 9.3261 \; mm \qquad r_A = 40 \; mm.$

$$\sigma_A = -\frac{M y_A}{A e r_A} = -\frac{(68)(0.0093261)}{(600 \times 10^{-6})(0.0006739)(0.04)} = -39.2 \; MPa$$

$-39.2 \; MPa$ ◄

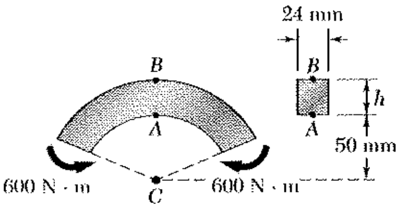
$y_B = 49.3261 - 60 = -10.6739 \qquad r_B = 60 \; mm$

$$\sigma_B = -\frac{M y_B}{A e r_B} = -\frac{(68)(-10.6739)10^{-3}}{(600 \times 10^{-6})(0.0006739)(0.06)} = 29.9 \; MPa$$

$29.9 \; MPa$ ◄

Problem 4.158

4.158 For the curved bar and loading shown, determine the stress at points *A* and *B* when *h* = 55 mm.



$h = 55 \text{ mm}, \qquad r_1 = 50 \text{ mm}, \qquad r_2 = 105 \text{ mm},$

$A = (24)(55) = 1.320 \times 10^3 \text{ mm}^2$

$R = \dfrac{h}{\ln \frac{r_2}{r_1}} = \dfrac{55}{\ln \frac{105}{50}} = 74.13025 \text{ mm}$

$\bar{r} = \frac{1}{2}(r_1 + r_2) = 77.5 \text{ mm}$

$e = \bar{r} - R = 3,36975 \text{ mm}$

$y_A = 74.13025 - 50 = 24.13025 \text{ mm} \qquad r_A = 50 \text{ mm}$

$$\sigma_A = -\frac{M y_A}{A e r_A} = -\frac{(600)(24.13025 \times 10^{-3})}{(1.320 \times 10^{-3})(3.36975 \times 10^{-3})(50 \times 10^{-3})} = -65.1 \times 10^6 \text{ Pa}$$

$\sigma_A = -65.1 \text{ MPa}$ ◄

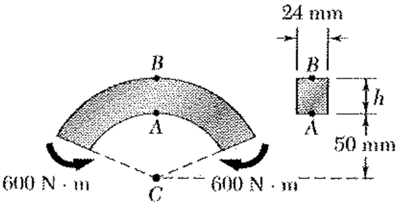
$y_B = 74.13025 - 105 = -30.86975 \text{ mm} \qquad r_B = 105 \text{ mm}$

$$\sigma_B = -\frac{M y_B}{A e r_B} = -\frac{(600)(-30.8697 \times 10^{-3})}{(1.320 \times 10^{-3})(3.36975 \times 10^{-3})(105 \times 10^{-3})} = 39.7 \times 10^6 \text{ Pa}$$

$\sigma_B = 39.7 \text{ MPa}$ ◄

Problem 4.159

4.159 For the curved bar and loading shown, determine the stress at point A when (*a*) $h = 50$ mm, (*b*) $h = 60$ mm.



(a) $h = 50\text{ mm}, \quad r_1 = 50\text{ mm}, \quad r_2 = 100\text{ mm}$

$$A = (24)(50) = 1.200\times10^{3}\text{ mm}$$

$$R = \frac{h}{\ln\frac{r_2}{r_1}} = \frac{50}{\ln\frac{100}{50}} = 72.13475\text{ mm}$$

$$\bar{r} = \tfrac{1}{2}(r_1 + r_2) = 75\text{ mm}$$

$$e = \bar{r} - R = 2.8652\text{ mm}$$

$$y_A = 72.13475 - 50 = 22.13475\text{ mm} \qquad r_A = 50\text{ mm}$$

$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(600)(22.13475\times10^{-3})}{(1.200\times10^{-3})(2.8652\times10^{-3})(50\times10^{-3})} = -77.3\times10^{6}\text{ Pa}$$

$$\sigma_A = -77.3\text{ MPa} \quad ◄$$

(b) $h = 60\text{ mm}, \quad r_1 = 50\text{ mm}, \quad r_2 = 110\text{ mm}, \quad A = (24)(60) = 1.440\times10^{3}\text{ mm}^2$

$$R = \frac{h}{\ln\frac{r_2}{r_1}} = \frac{60}{\ln\frac{110}{50}} = 76.09796\text{ mm}$$

$$\bar{r} = \tfrac{1}{2}(r_1 + r_2) = 80\text{ mm} \qquad e = \bar{r} - R = 3.90204\text{ mm}$$

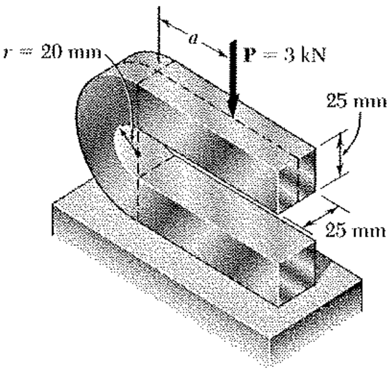
$$y_A = 76.09796 - 50 = 26.09796\text{ mm} \qquad r_A = 50\text{ mm}$$

$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(600)(26.09796\times10^{-3})}{(1.440\times10^{-3})(3.90204\times10^{-3})(50\times10^{-3})} = -55.7\times10^{6}\text{ Pa}$$

$$\sigma_A = -55.7\text{ MPa} \quad ◄$$

Problem 4.160

4.160 The curved portion of the bar shown has an inner radius of 20 mm. Knowing that the allowable stress in the bar is 150 MPa, determine the largest permissible distance *a* from the line of action of the 3-kN force to the vertical plane containing the center of curvature of the bar.



Reduce the internal forces transmitted across section AB to a force-couple system at the centroid of the section. The bending couple is

$$M = P(a+\bar{r})$$

For the rectangular section, the neutral axis for bending couple only lies

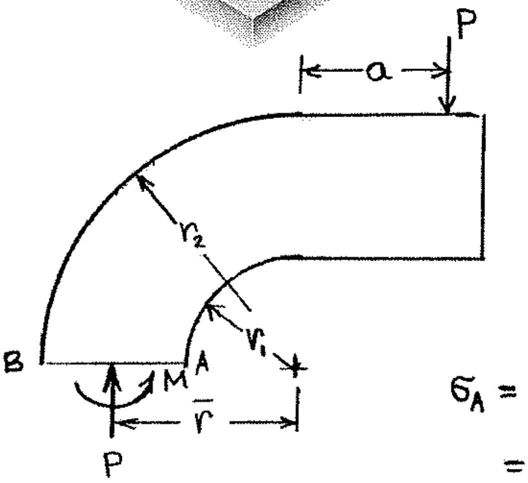
$$R = \frac{h}{\ln\frac{r_2}{r_1}} \qquad \text{Also} \quad e = \bar{r} - R$$

The maximum compressive stress occurs at point A. It is given by

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{P}{A} - \frac{P(a+\bar{r})y_A}{Aer_1}$$

$$= -K\frac{P}{A} \qquad \text{with} \quad y_A = R - r_1$$

Thus, $K = 1 + \frac{(a+\bar{r})(R-r_1)}{er_1}$



Data: $h = 25$ mm, $r_1 = 20$ mm, $r_2 = 45$ mm, $\bar{r} = 32.5$ mm

$$R = \frac{25}{\ln\frac{45}{20}} = 30.8288 \text{ mm}, \quad e = 32.5 - 30.8288 = 1.6712 \text{ mm}$$

$b = 25$ mm, $A = bh = (25)(25) = 625 \text{ mm}^2 = 625\times10^{-6} \text{ m}^2$

$R - r_1 = 10.8288$ mm

$P = 3\times10^3$ N·m $\qquad \sigma_A = -150\times10^6$ Pa

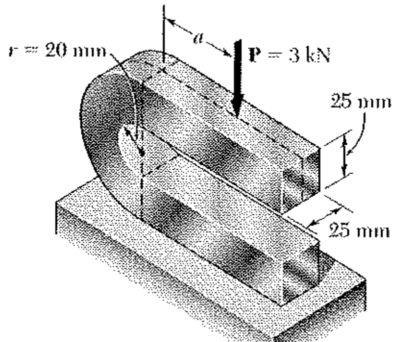
$$K = -\frac{\sigma_A A}{P} = -\frac{(-150\times10^6)(625\times10^{-6})}{3\times10^3} = 31.25$$

$$a+\bar{r} = \frac{(K-1)er_1}{R-r_1} = \frac{(30.25)(1.6712)(20)}{10.8288} = 93.37 \text{ mm}$$

$a = 93.37 - 32.5 \qquad a = 60.9$ mm ◄

Problem 4.161

4.161 The curved portion of the bar shown has an inner radius of 20 mm. Knowing that the line of action of the 3-kN force is located at a distance $a = 60$ mm from the vertical plane containing the center of curvature of the bar, determine the largest compressive stress in the bar.



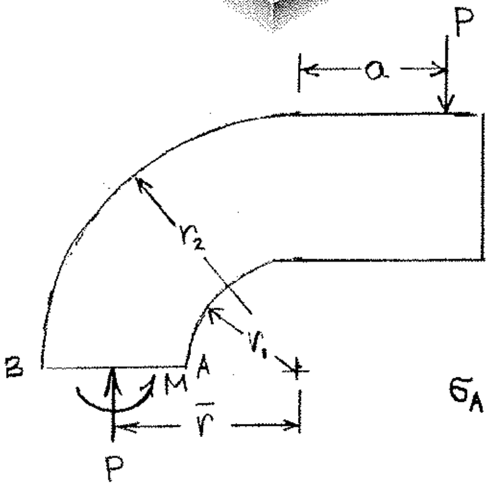
Reduce the internal forces transmitted across section AB to a force-couple system at the centroid of the section. The bending couple is

$$M = P(a + \bar{r})$$

For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln \frac{r_2}{r_1}}. \qquad \text{Also} \quad e = \bar{r} - R$$

The maximum compressive stress occurs at point A. It is given by



$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{P}{A} - \frac{P(a+\bar{r})y_A}{Aer_1}$$

$$= -K\frac{P}{A} \qquad \text{with } y_A = R - r_1$$

$$\text{Thus,} \quad K = 1 + \frac{(a+\bar{r})(R-r_1)}{er_1}$$

Data: $h = 25$ mm, $r_1 = 20$ mm, $r_2 = 45$ mm, $\bar{r} = 32.5$ mm

$$R = \frac{25}{\ln \frac{45}{20}} = 30.8288 \text{ mm}, \quad e = 32.5 - 30.8288 = 1.6712 \text{ mm}$$

$$b = 25 \text{ mm}, \quad A = bh = (25)(25) = 625 \text{ mm}^2 = 625 \times 10^{-6} \text{ m}^2$$

$$a = 60 \text{ mm}, \quad a + \bar{r} = 92.5 \text{ mm}, \quad R - r_1 = 10.8288 \text{ mm}$$

$$K = 1 + \frac{(92.5)(10.8288)}{(1.6712)(20)} = 30.968$$

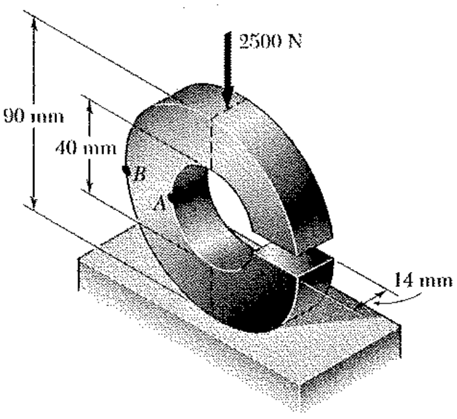
$$P = 3 \times 10^3 \text{ N}$$

$$\sigma_A = -\frac{KP}{A} = -\frac{(30.968)(3 \times 10^3)}{625 \times 10^{-6}} = -148.6 \times 10^6 \text{ Pa}$$

$$\sigma_A = -148.6 \text{ MPa}$$ ☚

Problem 4.162

4.162 For the split ring shown, determine the stress at (*a*) point *A*, (*b*) point *B*.



$r_1 = \frac{1}{2} 40 = 20 \text{ mm}$, $r_2 = \frac{1}{2}(90) = 45 \text{ mm}$

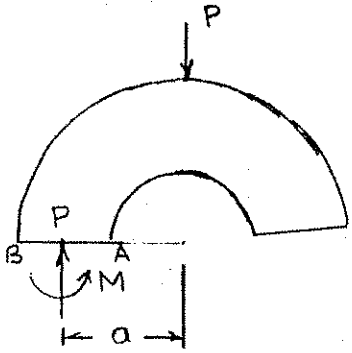
$h = r_2 - r_1 = 25 \text{ mm}$

$A = (14)(25) = 350 \text{ mm}^2$

$R = \dfrac{h}{\ln \frac{r_2}{r_1}} = \dfrac{25}{\ln \frac{45}{20}} = 30.8288 \text{ mm}$

$\bar{r} = \frac{1}{2}(r_1 + r_2) = 32.5 \text{ mm}$

$e = \bar{r} - R = 1.6712 \text{ mm}$



Reduce the internal forces transmitted across section AB to a force-couple system at the centroid of the cross section. The bending couple is

$M = Pa = P\bar{r}$

$= (2500)(32.5 \times 10^{-3}) = 81.25 \text{ N} \cdot \text{m}$

(a) <u>Point A:</u> $r_A = 20 \text{ mm}$

$y_A = 30.8288 - 20 = 10.8288 \text{ mm}$

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{AeR} = -\frac{2500}{350 \times 10^{-6}} - \frac{(81.25)(10.8288 \times 10^{-3})}{(350 \times 10^{-6})(1.6712 \times 10^{-3})(20 \times 10^{-3})}$$

$= -82.4 \times 10^6 \text{ Pa}$ $\sigma_A = -82.4 \text{ MPa}$ ◄

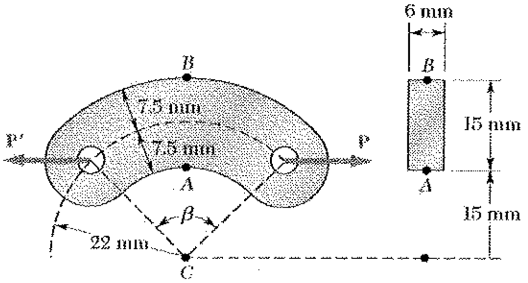
(b) <u>Point B:</u> $r_B = 45 \text{ mm}$ $y_B = 30.8288 - 45 = -14.1712 \text{ mm}$

$$\sigma_B = -\frac{P}{A} - \frac{My_B}{Aer_B} = -\frac{2500}{350 \times 10^{-6}} - \frac{(81.25)(-14.1712 \times 10^{-3})}{(350 \times 10^{-6})(1.6712 \times 10^{-3})(45 \times 10^{-3})}$$

$= 36.6 \times 10^6 \text{ Pa}$ $\sigma_B = 36.6 \text{ MPa}$ ◄

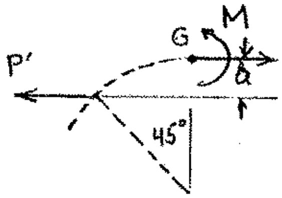
Problem 4.163

4.163 Steel links having the cross section shown are available with different central angles β. Knowing that the allowable stress is 100 MPa, determine the largest force **P** that can be applied to a link for which $\beta = 90°$.



Reduce section force to a force-couple system at G, the centroid of the cross section AB.

$$a = \bar{r}\left(1 - \cos\frac{\beta}{2}\right)$$



The bending couple is $M = -Pa$

For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln\frac{r_2}{r_1}}. \qquad \text{Also} \quad e = \bar{r} - R$$

At point A the tensile stress is

$$\sigma_A = \frac{P}{A} - \frac{My_A}{Aer_1} = \frac{P}{A} + \frac{Pay_A}{Aer_1} = \frac{P}{A}\left(1 + \frac{ay_A}{er_1}\right) = K\frac{P}{A}$$

where $K = 1 + \frac{ay_A}{er_1}$ and $y_A = R - r_1$

$$P = \frac{A\sigma_A}{K}$$

Data: $\bar{r} = 22\,\text{mm}$, $r_1 = 15\,\text{mm}$, $r_2 = 30\,\text{mm}$, $h = 15\,\text{mm}$, $b = 6\,\text{mm}$.

$A = (15)(6) = 90\,\text{mm}^2$, $\quad R = \frac{15}{\ln\frac{30}{15}} = 21.64\,\text{mm}$.

$e = 22 - 21.64 = 0.36\,\text{mm}$, $\quad y_A \doteq 21.64 - 15 = 6.64\,\text{mm}$.

$a = 22\,(1 - \cos 45°) = 6.44\,\text{mm}$

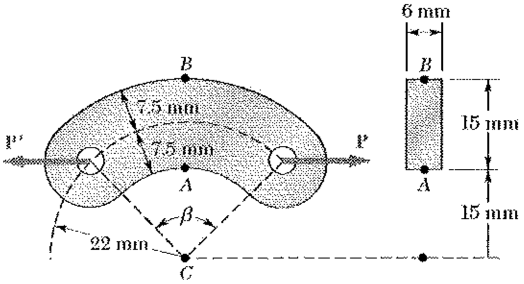
$$K = 1 + \frac{(6.44)(6.64)}{(0.36)(15)} = 8.92$$

$$P = \frac{(90\times10^{-6})(100\times10^{6})}{8.92} = 1009\text{ N} = 1\text{ kN}$$

Problem 4.164

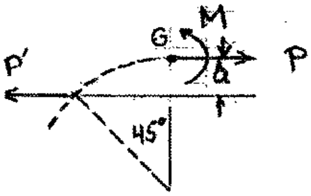
4.164 Solve Prob. 4.163, assuming that $\beta = 60°$.

4.163 Steel links having the cross section shown are available with different central angles β. Knowing that the allowable stress is 100 MPa, determine the largest force **P** that can be applied to a link for which $\beta = 90°$.



Reduce section force to a force-couple system at G, the centroid of the cross section AB.

$$a = \bar{r}\left(1 - \cos\frac{\beta}{2}\right)$$



The bending couple is $M = -Pa$

For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln\frac{r_2}{r_1}}. \qquad \text{Also} \quad e = \bar{r} - R$$

At point A the tensile stress is

$$\sigma_A = \frac{P}{A} + \frac{My_A}{Aer_1} = \frac{P}{A} + \frac{Pay_A}{Aer_1} = \frac{P}{A}\left(1 + \frac{ay_A}{er_1}\right) = K\frac{P}{A}$$

where $K = 1 + \frac{ay_A}{er_1}$ and $y_A = R - r_1$

$$P = \frac{A\sigma_A}{K}$$

Data: $\bar{r} = 22\text{mm}$, $r_1 = 15\text{mm}$, $r_2 = 30\text{mm}$, $h = 15\text{mm}$, $b = 6\text{mm}$.

$A = (6)(15) = 900\text{mm}^2$, $R = \frac{15}{\ln\frac{30}{15}} = 21.64\text{mm}$

$e = 22 - 21.64 = 0.36\text{mm}$ $\quad y_A = 21.64 - 15 = 6.64\text{ mm}$.

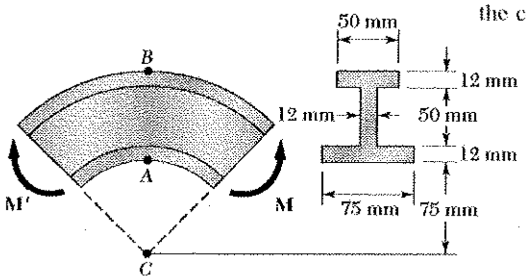
$a = 22(1 - \cos 30°) = 2.95\text{ mm}$.

$$K = 1 + \frac{(2.95)(6.64)}{(0.36)(15)} = 4.627$$

$$P = \frac{(900\times10^{-6})(100\times10^{6})}{4.627} = 1945\text{ N} = 1.945\text{ kN}.$$

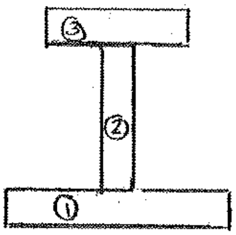
Problem 4.165

4.165 Three plates are welded together to form the curved beam shown. For the given loading, determine the distance e between the neutral axis and the centroid of the cross section.



$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r} dA} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A \bar{r}_i}{\Sigma A}$$



r: 75, 87, 137, 149.

part	b	h	A	$b\ln\frac{r_2}{r_1}$	$\bar{r}$	$A\bar{r}$
①	75	12	900	11.1315	81	72900
②	12	50	600	5.4489	112	67200
③	50	12	600	4.1983	143	85800
Σ			2100	20.7787		225900

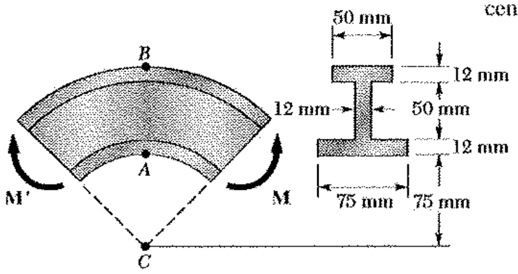
$R = \frac{2100}{20.7787} = 101.07$ mm, $\bar{r} = \frac{225900}{2100} = 107.57$

$e = \bar{r} - R = 6.5$ mm

6.5 mm ◄

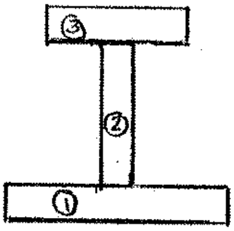
Problem 4.166

4.166 Three plates are welded together to form the curved beam shown. For $M = 900\ \text{N}\cdot\text{m}$, determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) the centroid of the cross section.



$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r} dA} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A \bar{r}_i}{\Sigma A}$$



r	part	b	h	A	$b \ln \frac{r_2}{r_1}$	$\bar{r}$	$A\bar{r}$
75	①	75	12	900	11.1315	81	72900
87	②	12	50	600	5.4489	112	67200
137	③	50	12	600	4.1983	143	85800
149.	Σ			2100	20.7787		225900

$R = \frac{2100}{20.7787} = 101.07\ mm$, $\bar{r} = \frac{225900}{2100} = 107.57\ mm$

$e = \bar{r} - R = 6.5\ mm$ $\qquad M = -900\ Nm.$

(a) $y_A = R - r_1 = 101.07 - 75 = 26.07\ mm.$

$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(-900)(0.02607)}{(2100\times10^{-6})(0.0065)(0.075)} = 22.9\ MPa$$ ◀

(b) $y_B = R - r_2 = 101.07 - 149 = -47.93$

$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(-900)(-0.04793)}{(2100\times10^{-6})(0.0065)(0.149)} = -21.2\ MPa.$$ ◀

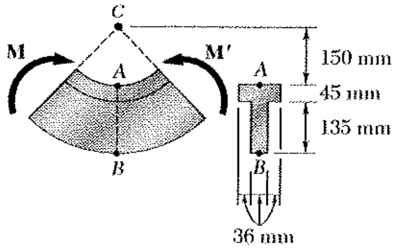
(c) $y_c = R - \bar{r} = -e =$

$$\sigma_c = -\frac{M y_c}{A e \bar{r}} = -\frac{Me}{Ae\bar{r}} = -\frac{M}{A\bar{r}} = -\frac{-900}{(2100\times10^{-6})(0.10757)}$$

$$= 4 MPa$$ ◀

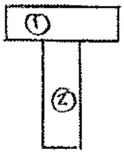
Problem 4.167

4.167 and 4.168 Knowing that $M = 20$ kN · m, determine the stress at (*a*) point *A*, (*b*) point *B*.



$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r} dA} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A_i \bar{r}_i}{\Sigma A_i}$$



r, mm	Part	b, mm	h, mm	A, mm^2	$b_i \ln \frac{r_{i+1}}{r_i}$, mm	$\bar{r}$, mm	$A\bar{r}$, mm^3
150							
	①	108	45	4860	28.3353	172.5	838.35×10^3
195							
	②	36	135	4860	18.9394	262.5	1275.75×10^3
330							
	Σ			9720	47.2747		2114.1×10^3

$$R = \frac{9720}{47.2747} = 205.606 \text{ mm} \qquad \bar{r} = \frac{2114.1 \times 10^3}{9720} = 217.5 \text{ mm}$$

$$e = \bar{r} - R = 11.894 \text{ mm.} \qquad M = 20 \times 10^3 \text{ N·m}$$

(a) $y_A = R - r_1 = 205.606 - 150 = 55.606$ mm

$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(20 \times 10^3)(55.606 \times 10^{-3})}{(9720 \times 10^{-6})(11.894 \times 10^{-3})(150 \times 10^{-3})}$$

$$= -64.1 \times 10^6 \text{ Pa}$$

$\sigma_A = -64.1$ MPa ◄

(b) $y_B = R - r_2 = 205.606 - 330 = -124.394$ mm

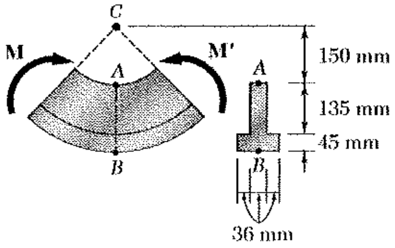
$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(20 \times 10^3)(-124.394 \times 10^{-3})}{(9720 \times 10^{-6})(11.894 \times 10^{-3})(330 \times 10^{-3})}$$

$$= 65.2 \times 10^6 \text{ Pa}$$

$\sigma_B = 65.2$ MPa ◄

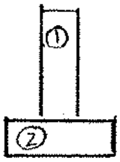
Problem 4.168

4.167 and 4.168 Knowing that $M = 20$ kN · m, determine the stress at (*a*) point *A*, (*b*) point *B*.



$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r} dA} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A_i \bar{r}_i}{\Sigma A_i}$$



r, mm: 150, 285, 330

	b_i, mm	h_i, mm	A, mm^2	$b_i \ln \frac{r_{i+1}}{r_i}$, mm	$\bar{r}$, mm	$A\bar{r}$, mm^3
①	36	135	4860	23.1067	217.5	1.05705×10^6
②	108	45	4860	15.8332	307.5	1.49445×10^6
Σ			9720	38.9399		2.5515×10^6

$$R = \frac{9720}{38.9399} = 249.615 \text{ mm}, \qquad \bar{r} = \frac{2.5515 \times 10^6}{9720} = 262.5 \text{ mm}$$

$$e = \bar{r} - R = 12.885 \text{ mm}, \qquad M = 20 \times 10^3 \text{ N}\cdot\text{m}$$

(a) $y_A = R - r_1 = 249.615 - 150 = 99.615$ mm

$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(20 \times 10^3)(99.615 \times 10^{-3})}{(9720 \times 10^{-6})(12.885 \times 10^{-3})(150 \times 10^{-3})}$$

$$= -106.1 \times 10^6 \text{ Pa}$$

$\sigma_A = -106.1$ MPa ◄

(b) $y_B = R - r_2 = 249.615 - 330 = -80.385$ mm

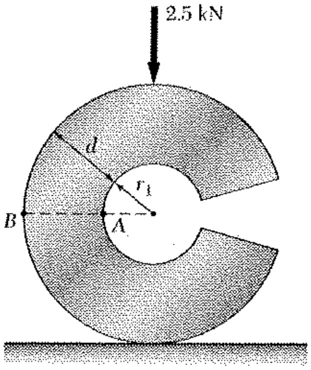
$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(20 \times 10^3)(-80.385 \times 10^{-3})}{(9720 \times 10^{-6})(12.885 \times 10^{-3})(330 \times 10^{-6})}$$

$$= 38.9 \times 10^6 \text{ Pa}$$

$\sigma_B = 38.9$ MPa ◄

Problem 4.169

4.169 The split ring shown has an inner radius $r_1 = 20$ mm and a *circular* cross section of diameter $d = 32$ mm. For the loading shown, determine the stress at (*a*) point *A*, (*b*) point *B*.



$c = \frac{1}{2}d = 16\text{mm} \qquad r_1 = 20 \text{ mm}, \quad r_2 = r_1 + d = 52 \text{ mm}$

$\bar{r} = r_1 + c = 36 \text{ mm}$

$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\,\right] = \frac{1}{2}\left[36 + \sqrt{36^2 - 16^2}\,\right]$

$= 34.1245 \text{ mn.}$

$e = \bar{r} - R = 1.8755 \text{ mm}$

$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(32)^2 = 804.25 \text{ mm}^2 = 804.25 \times 10^{-6} \text{ m}^2$

$P = 2.5 \times 10^3 \text{ N} \qquad M = P\bar{r} = (2.5 \times 10^3)(36 \times 10^{-3}) = 90 \text{ N}\cdot\text{m}$

(a) Point A : $y_A = R - r_1 = 34.1245 - 20 = 14.1245 \text{ mm}$

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(90)(14.1245 \times 10^{-3})}{(804.25 \times 10^{-6})(1.8755 \times 10^{-3})(20 \times 10^{-3})}$$

$$= -45.2 \times 10^6 \text{ Pa} \qquad \sigma_A = -45.2 \text{ MPa} ◄$$

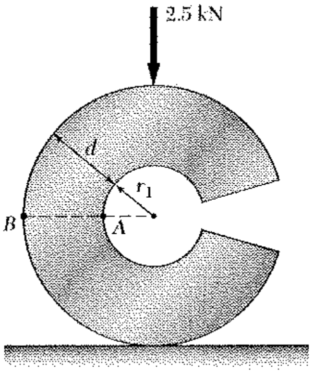
(b) Point B : $y_B = R - r_2 = 34.1245 - 52 = -17.8755 \text{ mm}$

$$\sigma_B = -\frac{P}{A} - \frac{My_B}{Aer_2} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(90)(-17.8755 \times 10^{-3})}{(804.25 \times 10^{-6})(1.8755 \times 10^{-3})(52 \times 10^{-3})}$$

$$= 17.40 \times 10^6 \text{ Pa} \qquad \sigma_B = 17.40 \text{ MPa} ◄$$

Problem 4.170

4.170 The split ring shown has an inner radius $r_1 = 16$ mm and a *circular* cross section of diameter $d = 32$ mm. For the loading shown, determine the stress at (*a*) point *A*, (*b*) point *B*.



$$c = \tfrac{1}{2}d = 16\text{ mm},\quad r_1 = 16\text{ mm},\quad r_2 = r_1 + d = 48\text{ mm}$$

$$\bar{r} = r_1 + c = 32\text{ mm}$$

$$R = \tfrac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \tfrac{1}{2}\left[32 + \sqrt{32^2 - 16^2}\right]$$

$$= 29.8564\text{ mm}$$

$$e = \bar{r} - R = 2.1436\text{ mm}$$

$$A = \tfrac{\pi}{4}d^2 = \tfrac{\pi}{4}(32)^2 = 804.25\text{ mm}^2 = 804.25\times10^{-6}\text{ m}^2$$

$P = 2.5\times10^3$ N $\qquad M = P\bar{r} = (2.5\times10^3)(32\times10^{-3}) = 80$ N·m

(a) Point A: $\quad y_A = R - r_1 = 29.8564 - 16 = 13.8564$ mm

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{2.5\times10^3}{804.25\times10^{-6}} - \frac{(80)(13.8564\times10^{-3})}{(804.25\times10^{-6})(2.1436\times10^{-3})(16\times10^{-3})}$$

$$= -43.3\times10^6\text{ Pa} \qquad \sigma_A = -43.3\text{ MPa} \blacktriangleleft$$

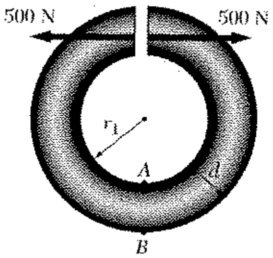
(b) Point B: $\quad y_B = R - r_2 = 29.8564 - 48 = -18.1436$ mm.

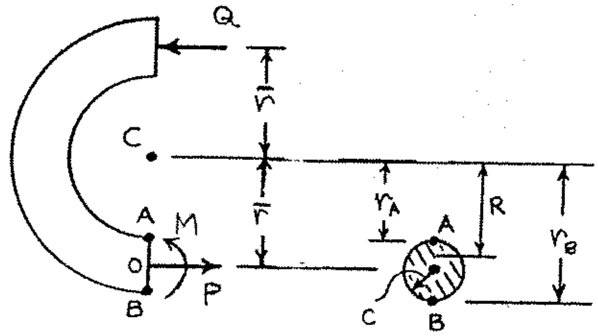
$$\sigma_B = -\frac{P}{A} - \frac{My_B}{Aer_2} = -\frac{2.5\times10^6}{804.25\times10^{-6}} - \frac{(80)(-18.1436\times10^{-3})}{(804.25\times10^{-6})(2.1436\times10^{-3})(48\times10^{-3})}$$

$$= 14.43\times10^6\text{ Pa} \qquad \sigma_B = 14.43\text{ MPa} \blacktriangleleft$$

Problem 4.171

4.171 The split ring shown has an inner radius $r_1 = 20$ mm and a *circular* cross section of diameter $d = 15$ mm. Knowing that each of the 500-N forces is applied at the centroid of the cross section, determine the stress at (*a*) point *A*, (*b*) point *B*.





$r_A = r_1 = 20$ mm

$r_B = r_A + d = 20 + 15 = 35$ mm

$\bar{r} = \frac{1}{2}(r_A + r_B) = 27.5$ mm $\qquad c = \frac{1}{2}d = 7.5$ mm

$A = \pi c^2 = \pi(7.5)^2 = 176.7$ mm^2 $\qquad$ for solid circular section

$$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \frac{1}{2}\left[27.5 + \sqrt{(27.5)^2 - (7.5)^2}\right] = 27 \text{ mm}$$

$e = \bar{r} - R = 27.5 - 27 = 0.5$ mm

$Q = 500$ N $\qquad \xrightarrow{+} \Sigma F_x = 0: \; P - Q = 0 \qquad P = 500$ N

$\circlearrowleft \Sigma M_o = 0: \; 2\bar{r}Q + M = 0 \qquad M = -2\bar{r}Q = -(2)(0.0275)(500) = -27.5$ N m

(a) $r = r_A = 20$ mm

$$\sigma_A = \frac{P}{A} + \frac{M(r_A - R)}{A e r_A} = \frac{500}{176.7\times10^{-6}} + \frac{(-27.5)(0.02 - 0.027)}{(176.7\times10^{-6})(0.0005)(0.02)}$$

$= 111.77 \times 10^6$ Pa

$\sigma_A = 111.8$ MPa ◀

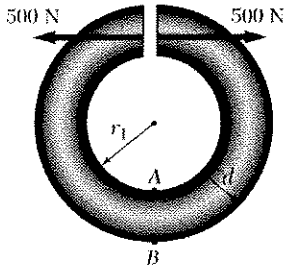
(b) $r = r_B = 35$ mm

$$\sigma_B = \frac{P}{A} + \frac{M(r_B - R)}{A e r_B} = \frac{500}{176.7\times10^{-6}} + \frac{(-27.5)(0.035 - 0.027)}{(176.7\times10^{-6})(0.0005)(0.035)}$$

$= -68.32 \times 10^6$

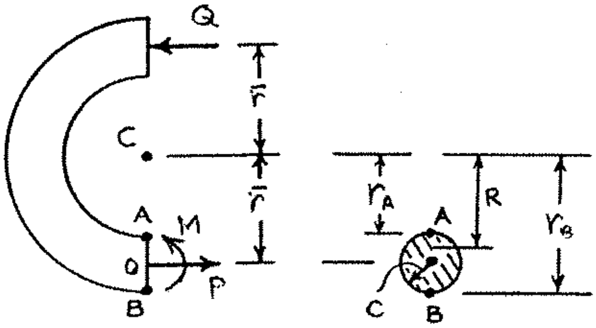
$\sigma_B = -68.3$ MPa ◀

Problem 4.172



4.172 Solve Prob. 4.171, assuming that the ring has an inner radius $r_1 = 15$ mm and a cross-sectional diameter $d = 20$ mm.

4.171 The split ring shown has an inner radius $r_1 = 20$ mm and a *circular* cross section of diameter $d = 15$ mm. Knowing that each of the 500-N forces is applied at the centroid of the cross section, determine the stress at (*a*) point *A*, (*b*) point *B*.



$r_A = r_1 = 15 mm$

$r_B = r_A + d = 15 + 20 = 35 mm$

$\bar{r} = \frac{1}{2}(r_A + r_B) = 25 mm$ $\qquad c = \frac{1}{2}d = 10 mm$

$A = \pi c^2 = \pi(10)^2 = 314.16 \; mm^2$ for solid circular section.

$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \frac{1}{2}\left[25 + \sqrt{(25)^2 - (10)^2}\right] = 24 \; mm$

$e = \bar{r} - R = 25 - 24 = 1 mm$

$Q = 500 \; N$ $\qquad \xrightarrow{+} \Sigma F_x = 0 \qquad P - Q = 0 \qquad P = 500 N$

$+\circlearrowleft \Sigma M_O = 0: \quad 2\bar{r}Q + M = 0 \qquad M = -2\bar{r}Q = -(2)(0.001)(500) = -1 \; Nm$

(a) $r = r_A = 15 mm$

$$\sigma_A = \frac{P}{A} + \frac{M(r_A - R)}{A e r_A} = \frac{500}{314.16 \times 10^{-6}} + \frac{(-1)(0.015 - 0.024)}{(314.16 \times 10^{-6})(0.001)(0.015)}$$

$= 3.501 \times 10^6 \; Pa$

$\sigma_A = 3.5 \; MPa$

(b) $r = r_B = 35 mm$

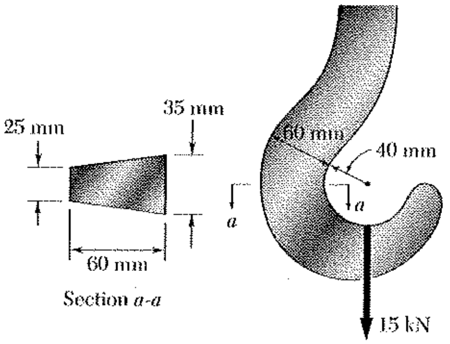
$$\sigma_B = \frac{P}{A} + \frac{M(r_B - R)}{A e r_B} = \frac{500}{314.16 \times 10^{-6}} + \frac{(-1)(0.035 - 0.024)}{(314.16 \times 10^{-6})(0.001)(0.035)}$$

$= -0.59 \times 10^6$

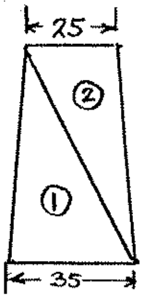
$\sigma_B = -0.6 \; MPa$

Problem 4.173

4.173 For the crane hook shown, determine the largest tensile stress in section *a-a*.



Locate centroid.



	A, mm^2	$\bar{r}$, mm	$A\bar{r}$, mm^3
①	1050	60	63×10^3
②	750	80	60×10^3
Σ	1800		103×10^3

$$\bar{r} = \frac{103 \times 10^3}{1800} = 68.333 \text{ mm.}$$

Force-couple system at centroid: $P = 15 \times 10^3$ N

$$M = -P\bar{r} = -(15 \times 10^3)(68.333 \times 10^{-3}) = -1.025 \times 10^3 \text{ N·m}$$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(60)^2(35 + 25)}{[(35)(100) - (25)(40)]\ln\frac{100}{40} - (60)(35+25)} = 63.878 \text{ mm.}$$

$$e = \bar{r} - R = 4.452 \text{ mm.}$$

Maximum tensile stress occurs at point A.

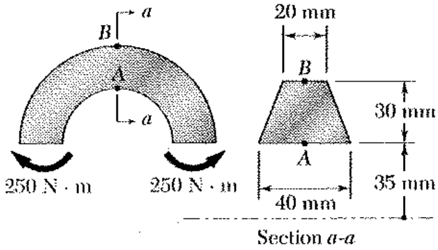
$$y_A = R - r_1 = 23.878 \text{ mm.}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{Aer_1} = \frac{15 \times 10^3}{1800 \times 10^{-6}} - \frac{-(1.025 \times 10^3)(23.878 \times 10^{-3})}{(1800 \times 10^{-6})(4.452 \times 10^{-3})(40 \times 10^{-3})}$$

$$= 84.7 \times 10^6 \text{ Pa} \qquad \sigma_A = 84.7 \text{ MPa} \blacktriangleleft$$

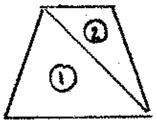
Problem 4.174

4.174 For the curved beam and loading shown, determine the stress at (*a*) point *A*, (*b*) point *B*.



Section *a-a*

Locate centroid.



	A, mm^2	$\bar{r}$, mm	$A\bar{r}$, mm^3
①	600	45	27×10^3
②	300	55	16.5×10^3
Σ	900		43.5×10^3

$$\bar{r} = \frac{43.5 \times 10^3}{900} = 48.333 \text{ mm.}$$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_2 - b_1)}$$

$$= \frac{(0.5)(30)^2(40+20)}{[(40)(65) - (20)(35)]\ln\frac{65}{35} - (30)(40 - 20)} = 46.8608 \text{ mm}$$

$$e = \bar{r} - R = 1.4725 \text{ mm} \qquad M = -250 \text{ N-m}$$

(a) $y_A = R - r_1 = 11.8608$ mm.

$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(-250)(11.8608 \times 10^{-3})}{(900 \times 10^{-6})(1.4725 \times 10^{-3})(35 \times 10^{-3})} = 63.9 \times 10^6 \text{ Pa}$$

$$\sigma_A = 63.9 \text{ MPa} \blacktriangleleft$$

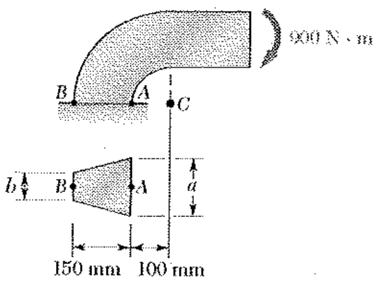
(b) $y_B = R - r_2 = -18.1392$ mm

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(-250)(-18.1392 \times 10^{-3})}{(900 \times 10^{-6})(1.4725 \times 10^{-3})(65 \times 10^{-3})} = -52.6 \times 10^6 \text{ Pa}$$

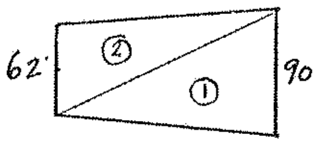
$$\sigma_B = -52.6 \text{ MPa} \blacktriangleleft$$

Problem 4.175

4.175 Knowing that the machine component shown has a trapezoidal cross section with $a = 90$ mm and $b = 62$ mm, determine the stress at (*a*) point *A*, (*b*) point *B*.



Locate centroid



	A, mm^2	$\bar{r}, mm$	$A\bar{r}, mm^3$
①	6750	150	1.0125×10^6
②	4650	200	0.93×10^6
Σ	11400		1942500

$$\bar{r} = \frac{1942500}{11400} = 170.4 \text{ mm}$$

$$R = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(b_1 r_2 - b_2 r_1) \ln \frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(150)^2 (90 + 62)}{[(90)(250) - (62)(100)] \ln \frac{250}{100} - (150)(90 - 62)} = 159.3 \text{ mm}$$

$e = \bar{r} - R = 11.1$ $\qquad$ $M = 9 \text{ kN.}$

(a) $y_A = R - r_1 = 159.3 - 100 = 59.3$ mm

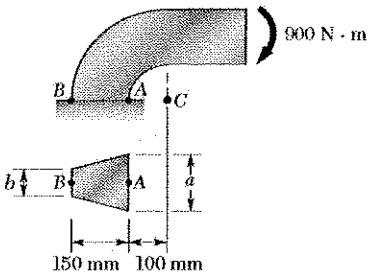
$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(9009(0.0593)}{(11400 \times 10^{-6})(0.0111)(0.1)} = -42.2 \text{ MPa.}$$

(b) $y_B = R - r_2 = 159.3 - 250 = -90.7$ mm.

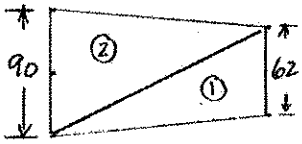
$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(9000)(-0.0907)}{(11400 \times 10^{-6})(0.0111)(0.25)} = 25.8 \text{ MPa}$$

Problem 4.176

4.176 Knowing that the machine component shown has a trapezoidal cross section with $a = 62$ mm and $b = 90$ mm, determine the stress at (*a*) point *A*, (*b*) point *B*.



Locate centroid



	A, in^2	$\bar{r}$, in.	$A\bar{r}$, in^3
①	4650	150	697500
②	6750	200	1.35×10^6
Σ	11400		2047500

$$\bar{r} = \frac{2047500}{11400} = 179.6 \text{ mm}$$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(150)^2(62+90)}{[(62)(250) - (90)(100)]\ln\frac{250}{100} - (150)(62-90)} = 168.4 \text{ mm}$$

$e = \bar{r} - R = 11.2$ mm $\qquad M = 9$ kN

(a) $y_A = R - r_1 = 68.4$ mm

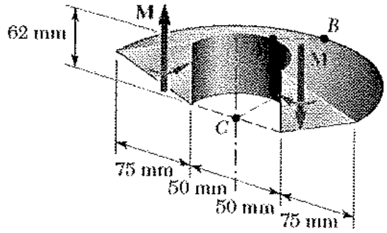
$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(9000)(0.0684)}{(11400 \times 10^{-6})(0.0112)(0.1)} = -48.2 \text{ MPa}$$

(b) $y_B = R - r_2 = -81.6$ mm

$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(9000)(-0.0816)}{(11400 \times 10^{-6})(0.0112)(0.25)} = 23 \text{ MPa}$$

Problem 4.177

4.177 and 4.178 Knowing that $M = 560$ N · m, determine the stress at (*a*) point *A*, (*b*) point *B*.



$A = \frac{1}{2} bh = \frac{1}{2}(62)(75) = 2325 \text{ mm}^2$

$\bar{r} = 50 + 25 = 75 \text{ mm}$

$b_1 = 62 \text{ mm}, \ r_1 = 50 \text{ mm}, \ b_2 = 0; \ r_2 = 125 \text{ mm}$

Use formula for trapezoid with $b_2 = 0$.

$$R = \frac{\frac{1}{2}h^2 (b_1 + b_2)}{(b_1 r_2 - b_2 r_1) \ln \frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(75)^2(62+0)}{[(62)(125) - (0)(50)] \ln \frac{125}{50} - 75(62-0)} = 71.14 \text{ mm}$$

$e = \bar{r} - R = 3.86 \text{ mm}$ $\qquad M = 560 \text{ Nm}$

(a) $y_A = R - r_1 = 21.14 \text{ mm}.$

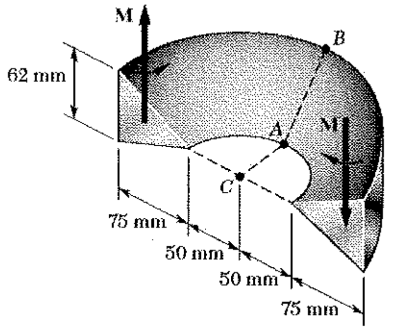
$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(560)(0.02114)}{(2325 \times 10^{-6})(0.00386)(0.05)} = -26.4 \text{ MPa}.$$

(b) $y_B = R - r_2 = -53.86 \text{ mm}$

$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(560)(-0.05386)}{(2325 \times 10^{-6})(0.00386)(0.125)} = 26.9 \text{ MPa}$$

Problem 4.178

4.177 and 4.178 Knowing that $M = 560$ N · m, determine the stress at (*a*) point *A*, (*b*) point *B*.



$$A = \tfrac{1}{2}(62)(75) = 2325 \text{ mm}^2.$$

$$\bar{r} = 50 + 50 = 100 \text{ mm}$$

$$b_1 = 0, \quad r_1 = 50\text{mm}, \quad b_2 = 62\text{mm} \quad r_2 = 125 \text{ mm}.$$

Use formula for trapezoid with $b_1 = 0$.

$$R = \frac{\tfrac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(75)^2(0+62)}{[(0)(125) - (62)(50)]\ln\frac{125}{50} - 75(0-62)} = 96.4 \text{ mm}$$

$$e = \bar{r} - R = 3.6 \text{ mm} \qquad M = 560 \text{ Nm}.$$

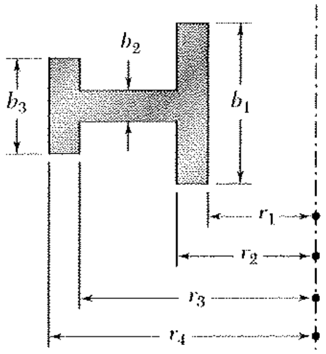
(a) $y_A = R - r_1 = 46.4$ mm

$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(560)(0.0464)}{(2325\times10^{-6})(0.0036)(0.05)} = -62.1 \text{ MPa}. \quad ◄$$

(b) $y_B = R - r_2 = -28.6$ mm

$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(560)(-0.0286)}{(2325\times10^{-6})(0.0036)(0.125)} = 15.3 \text{ MPa}. \quad ◄$$

Problem 4.179



4.179 Show that if the cross section of a curved beam consists of two or more rectangles, the radius R of the neutral surface can be expressed as

$$R = \frac{A}{\ln\left[\left(\frac{r_2}{r_1}\right)^{b_1}\left(\frac{r_3}{r_2}\right)^{b_2}\left(\frac{r_4}{r_3}\right)^{b_3}\right]}$$

where A is the total area of the cross section.

$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r} dA} = \frac{A}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$= \frac{A}{\Sigma \ln \left(\frac{r_{i+1}}{r_i}\right)^{b_i}} = \frac{A}{\ln\left[\left(\frac{r_2}{r_1}\right)^{b_1}\left(\frac{r_3}{r_2}\right)^{b_2}\left(\frac{r_4}{r_3}\right)^{b_3}\right]}$$

Note that for each rectangle $\int \frac{1}{r} dA = \int_{r_i}^{r_{i+1}} b_i \frac{dr}{r}$

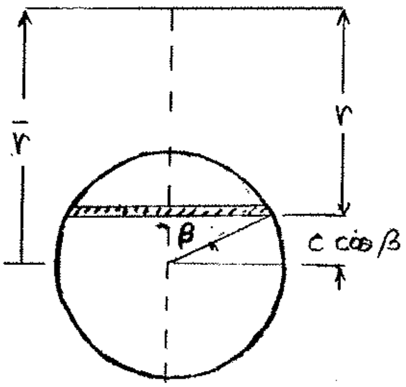
$$= b_i \int_{r_i}^{r_{i+1}} \frac{dr}{r} = b_i \ln \frac{r_{i+1}}{r_i}$$

Problem 4.180

4.180 through 4.182 Using Eq. (4.66), derive the expression for *R* given in Fig. 4.79 for

***4.180** A circular cross section.

Use polar coordinate β as shown.



$$\text{width: } w = 2c\sin\beta$$

$$r = \bar{r} - c\cos\beta$$

$$dr = c\sin\beta\, d\beta$$

$$dA = w\, dr = 2c^2\sin^2\beta\, d\beta$$

$$\int\frac{dA}{r} = \int_0^{\pi}\frac{2c^2\sin^2\beta}{\bar{r} - c\cos\beta}\, d\beta$$

$$\int\frac{dA}{r} = \int_0^{\pi}\frac{c^2(1-\cos^2\beta)}{\bar{r} - c\cos\beta}\, d\beta = 2\int_0^{\pi}\frac{\bar{r}^2 - c^2\cos^2\beta - (\bar{r}^2 - c^2)}{\bar{r} - c\cos\beta}\, d\beta$$

$$= 2\int_0^{\pi}(\bar{r} + c\cos\beta)\, d\beta - 2(\bar{r}^2 - c^2)\int_0^{\pi}\frac{dr}{\bar{r} - c\cos\beta}$$

$$= 2\bar{r}\,\beta\Big|_0^{\pi} + 2c\sin\beta\Big|_0^{\pi}$$

$$- 2(\bar{r}^2 - c^2)\frac{2}{\sqrt{\bar{r}^2 - c^2}}\tan^{-1}\frac{\sqrt{\bar{r}^2 - c^2}\,\tan(\tfrac{1}{2}\beta)}{\bar{r} + c}\Bigg|_0^{\pi}$$

$$= 2\bar{r}(\pi - 0) + 2c(0 - 0) - 4\sqrt{\bar{r}^2 - c^2}\cdot\left(\frac{\pi}{2} - 0\right)$$

$$2\pi\bar{r} - 2\pi\sqrt{\bar{r}^2 - c^2}$$

$$A = \pi c^2$$

$$R = \frac{A}{\int\frac{dA}{r}} = \frac{\pi c^2}{2\pi\bar{r} - 2\pi\sqrt{\bar{r}^2 - c^2}}$$

$$= \frac{1}{2}\,\frac{c^2}{\bar{r} - \sqrt{\bar{r}^2 - c^2}}\cdot\frac{\bar{r} + \sqrt{\bar{r}^2 - c^2}}{\bar{r} + \sqrt{\bar{r}^2 - c^2}}$$

$$= \frac{1}{2}\,\frac{c^2(\bar{r} + \sqrt{\bar{r}^2 - c^2})}{\bar{r}^2 - (\bar{r}^2 - c^2)} = \frac{1}{2}\,\frac{c^2(\bar{r} + \sqrt{\bar{r}^2 - c^2})}{c^2}$$

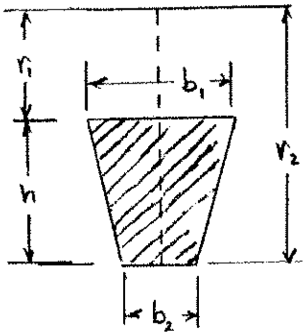
$$= \frac{1}{2}\left(\bar{r} + \sqrt{\bar{r}^2 - c^2}\right)$$

Problem 4.181

4.180 through 4.182 Using Eq. (4.66), derive the expression for *R* given in Fig. 4.79 for

4.181 A trapezoidal cross section.

The section width w varies linearly with r.



$$w = C_0 + C_1 r$$

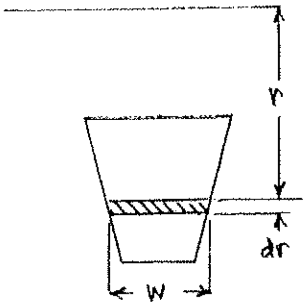
$w = b_1$ at $r = r_1$ and $w = b_2$ at $r = r_2$

$$b_1 = C_0 + C_1 r_1$$

$$b_2 = C_0 + C_1 r_2$$

$$b_1 - b_2 = C_1 (r_1 - r_2) = -C_1 h$$

$$C_1 = -\frac{b_1 - b_2}{h}$$



$$r_2 b_1 - r_1 b_2 = (r_2 - r_1) C_0 = h C_0$$

$$C_0 = \frac{r_2 b_1 - r_1 b_2}{h}$$

$$\int \frac{dA}{r} = \int_{r_1}^{r_2} \frac{w}{r} dr = \int_{r_1}^{r_2} \frac{C_0 + C_1 r}{r} dr$$

$$= C_0 \ln r \Big|_{r_1}^{r_2} + C_1 r \Big|_{r_1}^{r_2}$$

$$= C_0 \ln \frac{r_2}{r_1} + C_1 (r_2 - r_1)$$

$$= \frac{r_2 b_1 - r_1 b_2}{h} \ln \frac{r_2}{r_1} - \frac{b_1 - b_2}{h} h$$

$$= \frac{r_2 b_1 - r_1 b_2}{h} \ln \frac{r_2}{r_1} - (b_1 - b_2)$$

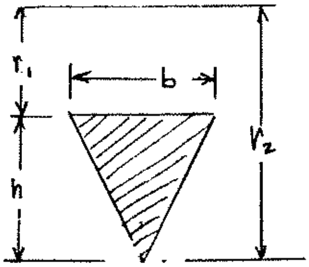
$$A = \frac{1}{2}(b_1 + b_2) h$$

$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(r_2 b_1 - r_1 b_2) \ln \frac{r_2}{r_1} - h (b_1 - b_2)}$$

Problem 4.182

4.180 through 4.182 Using Eq. (4.66), derive the expression for *R* given in Fig. 4.79 for

4.182 A triangular cross section.



The section width w varies linearly with r.

$$w = c_0 + c_1 r$$

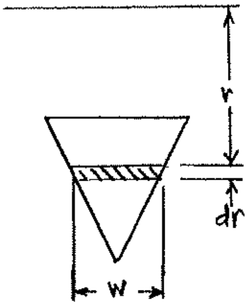
$w = b$ at $r = r_1$ and $w = 0$ at $r = r_2$

$$b = c_0 + c_1 r_1$$

$$0 = c_0 + c_1 r_2$$

$$b = c_1(r_1 - r_2) = -c_1 h$$

$$c_1 = -\frac{b}{h} \quad \text{and} \quad c_0 = -c_1 r_2 = \frac{b r_2}{h}$$



$$\int \frac{dA}{r} = \int_{r_1}^{r_2} \frac{w}{r} dr = \int_{r_1}^{r_2} \frac{c_0 + c_1 r}{r} dr$$

$$= c_0 \ln r \Big|_{r_1}^{r_2} + c_1 r \Big|_{r_1}^{r_2}$$

$$= c_0 \ln \frac{r_2}{r_1} + c_1(r_2 - r_1)$$

$$= \frac{b r_2}{h} \ln \frac{r_2}{r_1} - \frac{b}{h} h$$

$$= \frac{b r_2}{h} \ln \frac{r_2}{r_1} - b = b\left(\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1\right)$$

$$A = \frac{1}{2} b h$$

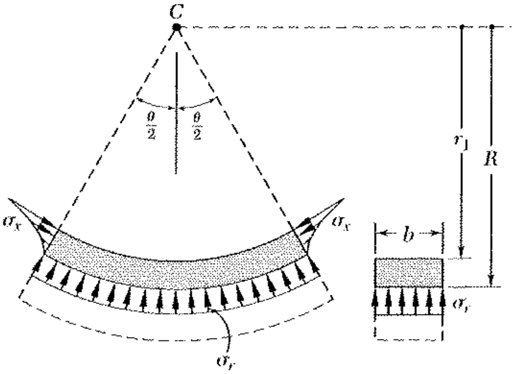
$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\frac{1}{2} b h}{b\left(\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1\right)} = \frac{\frac{1}{2} h}{\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1}$$

Problem 4.183

***4.183** For a curved bar of rectangular cross section subjected to a bending couple **M**, show that the radial stress at the neutral surface is

$$\sigma_r = \frac{M}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)$$

and compute the value of σ_r for the curved bar of Examples 4.10 and 4.11. (*Hint*: consider the free-body diagram of the portion of the beam located above the neutral surface.)



At radial distance r

$$\sigma_\theta = \frac{M(r-R)}{Aer} = \frac{M}{Ae} - \frac{MR}{Aer}$$

For portion above the neutral axis, the resultant force is

$$H = \int \sigma_\theta \, dA = \int_{r_1}^{R} \sigma_\theta \, b \, dr$$

$$= \frac{Mb}{Ae}\int_{r_1}^{R} dr - \frac{MRb}{Ae}\int_{r_1}^{2}\frac{dr}{r}$$

$$= \frac{Mb}{Ae}(R - r_1) - \frac{MRb}{Ae}\ln\frac{R}{r_1} \quad = \frac{MbR}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)$$

Resultant of σ_r:

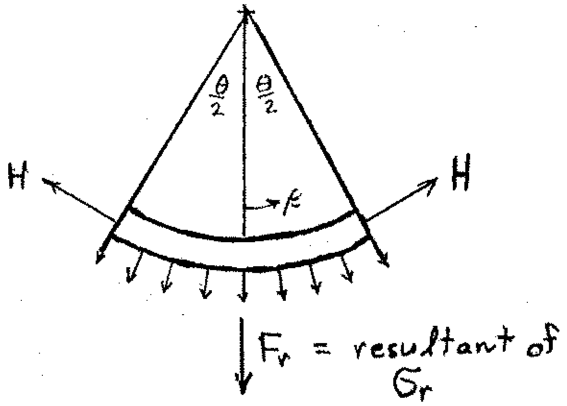
$$F_r = \int \sigma_r \cos\beta \, dA$$

$$= \int_{-\frac{\theta}{2}}^{\frac{\theta}{2}} \sigma_r \cos\beta \; b \, R d\beta$$

$$= \sigma_r bR \int_{-\frac{\theta}{2}}^{\frac{\theta}{2}} \cos\beta \, d\beta$$

$$= \sigma_r bR \; \sin\beta \Big|_{-\frac{\theta}{2}}^{\frac{\theta}{2}}$$

$$= 2\sigma_r bR \sin\frac{\theta}{2}$$



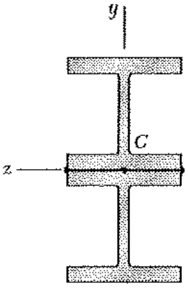
For equilibrium: $\quad F_r - 2H\sin\frac{\theta}{2} = 0$

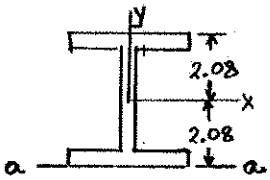
$$2\sigma_r bR \sin\frac{\theta}{2} - 2\frac{MbR}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)\sin\frac{\theta}{2} = 0$$

$$\sigma_r = \frac{M}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right) \quad ◄$$

Problem 4.184

4.184 and 4.185 Two W100 × 19.3 rolled sections are welded together as shown. Knowing that for the steel alloy used $\sigma_Y = 250$ MPa and $\sigma_U = 400$ MPa and using a factor of safety of 3.0, determine the largest couple that can be applied when the assembly is bent about the z axis.





Properties of W100x19.3 rolled section
See Appendix B

Area = 2480 mm^2 Depth = 106 mm.
$I_x = + 4.77 \times 10^6 \ mm^4$

For one rolled section, moment of inertia about axis a-a is

$$I_a = I_x + Ad^2 = 4.77 \times 10^6 + (2480)(52)^2 = 11.476 \times 10^6 \ mm^4.$$

For both sections $I_z = 2I_a = 22.95 \times 10^6 \ mm^4.$

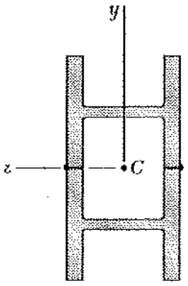
c = depth = 106 mm

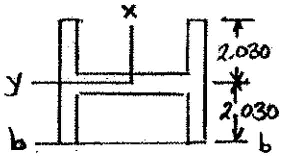
$$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{400}{3.0} = 133.33 \ MPa \qquad \sigma = \frac{Mc}{I}$$

$$M_{all} = \frac{\sigma_{all} I}{c} = \frac{(133.33 \times 10^6)(22.95 \times 10^{-6})}{0.106} = 28.9 \ kNm$$ ◄

Problem 4.185

4.184 and 4.185 Two W100 × 19.3 rolled sections are welded together as shown. Knowing that for the steel alloy used $\sigma_Y = 250$ MPa and $\sigma_U = 400$ MPa and using a factor of safety of 3.0, determine the largest couple that can be applied when the assembly is bent about the z axis.





Properties of W100x19.3 rolled section
See Appendix B

Area = 2480 mm^2 Width = 103 mm
$I_y = 1.61 \times 10^6 \ mm^4$

For one rolled section, moment of inertia about axis b-b is

$$I_b = I_y + Ad^2 = 1.61 \times 10^6 + (2480)(51.5)^2 = 8187580 \ mm^4$$

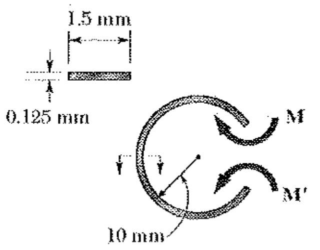
For both sections $I_z = 2I_b = 16.375 \times 10^6 \ mm^4.$

c = width = 103 mm.

$$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{400}{3.0} = 133.33 \ MPa \qquad \sigma = \frac{Mc}{I}$$

$$M_{all} = \frac{\sigma_{all} I}{c} = \frac{(133.33 \times 10^6)(16.375 \times 10^{-6})}{0.103} = 21.2 \ kNm.$$ ◄

Problem 4.186



4.186 It is observed that a thin steel strip of 1.5 mm width can be bent into a circle of 10-mm diameter without any resulting permanent deformation. Knowing that $E = 200$ GPa, determine (*a*) the maximum stress in the bent strip, (*b*) the magnitude of the couples required to bend the strip.

$$I = \frac{1}{12} b h^3 = \frac{1}{12}(1.5)(0.125)^3 = 2.44 \times 10^{-4}$$

$$\rho = \frac{1}{2} D = \frac{1}{2}(20) = 10 \text{ mm}$$

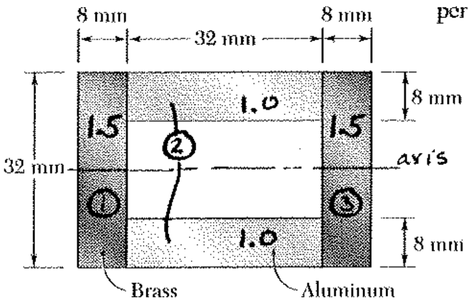
$$c = \frac{1}{2} h = 0.0625 \text{ mm}$$

(a) $$\sigma_{max} = \frac{Ec}{\rho} = \frac{(200 \times 10^9)(0.0625 \times 10^{-3})}{0.01} = 1250 \text{ MPa}$$

(b) $$M = \frac{EI}{\rho} = \frac{(200 \times 10^9)(2.44 \times 10^{-16})}{0.01} = 0.00488 \text{ Nm}.$$

Problem 4.187

4.187 A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.



	Aluminum	**Brass**
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

Use aluminum as the reference material.

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the sketch.

For the tranformed section,

$I_1 = \frac{n_1}{12} b_1 h_1^3 = \frac{1.5}{12}(8)(32)^3 = 32.768 \times 10^3 \text{ mm}^4$

$I_2 = \frac{n_2}{12} b_2 (H_2^3 - h_2^3) = \frac{1.0}{12}(32)(32^3 - 16^3) = 76.459 \times 10^3 \text{ mm}^4$

$I_3 = I_1 = 32.768 \times 10^3 \text{ mm}^4$

$I = I_1 + I_2 + I_3 = 141.995 \times 10^3 \text{ mm}^4 = 141.995 \times 10^{-9} \text{ m}^4$

$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \left|\frac{\sigma I}{ny}\right|$

Aluminum: $n = 1.0$, $|y| = 16 \text{ mm} = 0.016 \text{ m}$, $\sigma = 100 \times 10^6$ Pa

$$M = \frac{(100 \times 10^6)(141.995 \times 10^{-9})}{(1.0)(0.016)} = 887.47 \text{ N}\cdot\text{m}$$

Brass: $n = 1.5$, $|y| = 16 \text{ mm} = 0.016 \text{ m}$, $\sigma = 160 \times 10^6$ Pa

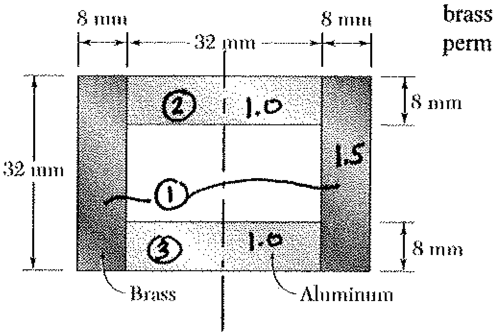
$$M = \frac{(160 \times 10^6)(141.995 \times 10^{-9})}{(1.5)(0.016)} = 946.63 \text{ N}\cdot\text{m}$$

Choose the smaller value. $M = 887 \text{ N}\cdot\text{m}$ ◄

Problem 4.188

4.188 For the composite bar of Prob. 4.187, determine the largest permissible bending moment when the bar is bent about a vertical axis.

4.187 A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.



	Aluminum	**Brass**
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

Use aluminum as the reference material.

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n and shown on the sketch.

For the transformed section,

$$I_1 = \frac{n_1}{12} h_1 (B_1^3 - b_1^3) = \frac{1.5}{12}(32)(48^3 - 32^3) = 311.296 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} h_2 b_2^3 = \frac{1.0}{12}(8)(32)^3 = 21.8453 \times 10^3 \text{ mm}^4$$

$$I_3 = I_2 = 21.8453 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 354.99 \times 10^3 \text{ mm}^4 = 354.99 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \left|\frac{\sigma I}{ny}\right|$$

Aluminum: $n = 1.0$, $|y| = 16 \text{ mm} = 0.016 \text{ m}$ $\sigma = 100 \times 10^6 \text{ Pa}$

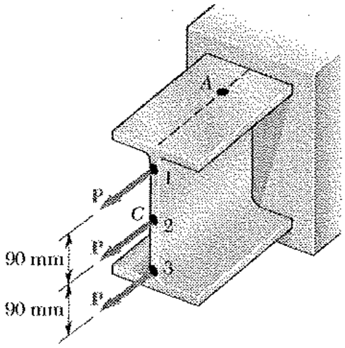
$$M = \frac{(100 \times 10^6)(354.99 \times 10^{-9})}{(1.0)(0.016)} = 2.2187 \times 10^3 \text{ N·m}$$

Brass: $n = 1.5$ $|y| = 24 \text{ mm} = 0.024 \text{ m}$ $\sigma = 160 \times 10^6 \text{ Pa}$

$$M = \frac{(160 \times 10^6)(354.99 \times 10^{-9})}{(1.5)(0.024)} = 1.57773 \times 10^3 \text{ N·m}$$

Choose the smaller value. $M = 1.57773 \times 10^3$ N·m $M = 1.578$ kN·m ◄

Problem 4.189



4.189 As many as three axial loads each of magnitude $P = 40$ kN can be applied to the end of a W200 × 31.3 rolled-steel shape. Determine the stress at point A, (a) for the loading shown, (b) if loads are applied at points 1 and 2 only.

For W200×31.3 Appendix C gives

$A = 4000\ mm^2 \quad d = 210\ mm, \quad I_x = 31.4 \times 10^6.$

At point A $\quad y = \frac{1}{2} d = 105$ mm.

$$\sigma = \frac{F}{A} - \frac{My}{I}$$

(a) Centric loading: $F = 120$ kN, $M = 0$

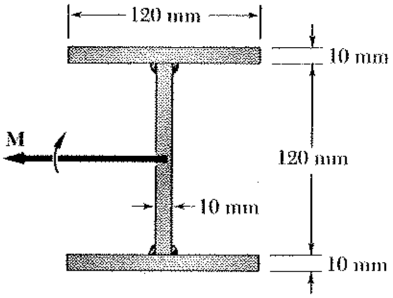
$$\sigma = \frac{120\,000}{4000 \times 10^{-6}} \qquad \sigma = 30\ MPa. \blacktriangleleft$$

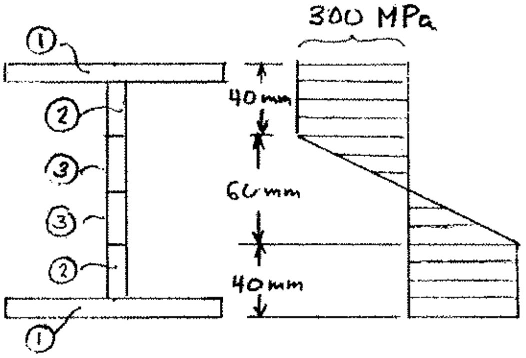
(b) Eccentric loading: $F = 2P = 80$ kN $\quad M = -(40)(0.09) = 3.6$ kNm

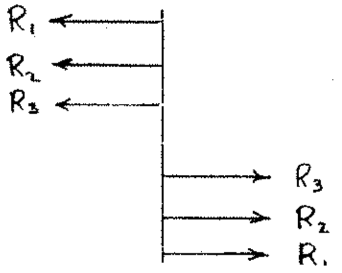
$$\sigma = \frac{80 \times 10^3}{4 \times 10^{-3}} - \frac{(-3.6 \times 10^3)(0.105)}{31.4 \times 10^{-6}} \qquad \sigma = 32\ MPa \blacktriangleleft$$

Problem 4.190

4.190 Three 120 × 10-mm steel plates have been welded together to form the beam shown. Assuming that the steel is elastoplastic with $E = 200$ GPa and $\sigma_Y = 300$ MPa, determine (*a*) the bending moment for which the plastic zones at the top and bottom of the beam are 40 mm thick, (*b*) the corresponding radius of curvature of the beam.







$A_1 = (120)(10) = 1200 \text{ mm}^2$ $\quad R_1 = \sigma_Y A_1 = (300\times10^6)(1200\times10^{-6}) = 360\times10^3$ N

$A_2 = (30)(10) = 300 \text{ mm}^2$ $\quad R_2 = \sigma_Y A_2 = (300\times10^6)(300\times10^{-6}) = 90\times10^3$ N

$A_3 = (30)(10) = 300 \text{ mm}^2$ $\quad R_3 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(300\times10^6)(300\times10^{-6})$

$= 45\times10^3$ N

$y_1 = 65 \text{ mm} = 65\times10^{-3}$ m

$y_2 = 45 \text{ mm} = 45\times10^{-3}$ m

$y_3 = 20 \text{ mm} = 20\times10^{-3}$ m

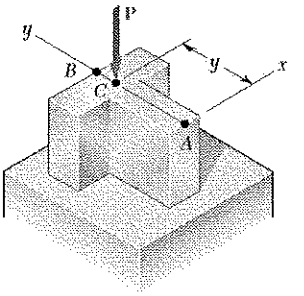
(a) $M = 2(R_1y_1 + R_2y_2 + R_3y_3) = 2\{(360)(65) + (90)(45) + (45)(20)\}$

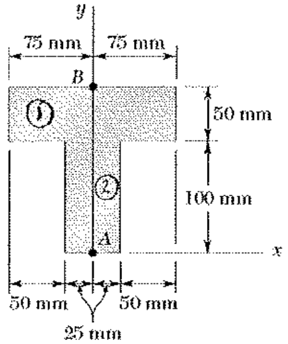
$= 56.7\times10^3$ N·m $= 56.7$ kN·m ◄

(b) $\frac{y_Y}{\rho} = \frac{\sigma_Y}{E}$ $\quad \rho = \frac{Ey_Y}{\sigma_Y} = \frac{(200\times10^9)(30\times10^{-3})}{300\times10^6} = 20$ m ◄

Problem 4.191

4.191 A vertical force **P** of magnitude 80 kN is applied at a point C located on the axis of symmetry of the cross section of a short column. Knowing that $y = 125$ mm, determine (a) the stress at point A, (b) the stress at point B, (c) the location of the neutral axis.





Locate centroid

Part	A, mm²	$\bar{y}$, mm	$A\bar{y}$, mm³
①	7500	125	937500
②	5000	50	250000
Σ	12500		1187500

$$\bar{y} = \frac{\Sigma A\bar{y}_i}{\Sigma A_i} = \frac{1187500}{12500} = 95 \text{ mm.}$$

Eccentricity of load $e = 125 - 95 = 30$ mm

$$I_1 = \frac{1}{12}(150)(50)^3 + (7500)(30)^2 = 8.3125 \times 10^6 \text{ mm}^4.$$

$$I_2 = \frac{1}{12}(50)(100)^3 + (5000)(45)^2 = 14.2917 \times 10^6 \text{ mm}^4.$$

$$I = I_1 + I_2 = 22.6 \times 10^6 \text{ mm}^4.$$

(a) Stress at A $c_A = 95$ mm.

$$\sigma_A = -\frac{P}{A} + \frac{Pec_A}{I} = \frac{80000}{12500 \times 10^{-6}} + \frac{(80000)(0.03)(0.095)}{22.6 \times 10^{-6}} = 3.69 \text{ MPa.}$$ ◄

(b) Stress at B $c_B = 150 - 95 = 55$ mm.

$$\sigma_B = -\frac{P}{A} - \frac{Pec_B}{I} = \frac{80000}{12500 \times 10^{-6}} - \frac{(80000)(0.03)(0.055)}{22.6 \times 10^{-6}} = -12.24 \text{ MPa}$$ ◄

(c) Location of neutral axis: $\sigma = 0$

$$\sigma = -\frac{P}{A} + \frac{Pea}{I} = 0 \quad \therefore \quad \frac{ea}{I} = \frac{1}{A}$$

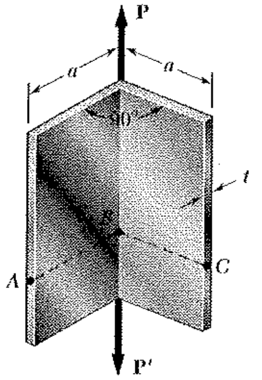
$$a = \frac{I}{Ae} = \frac{22.6 \times 10^6}{(12500)(30)} = 60.3 \text{ mm.}$$

Neutral axis lies 60.3 below centroid or 95 − 60.3
= 34.7 mm above point A.

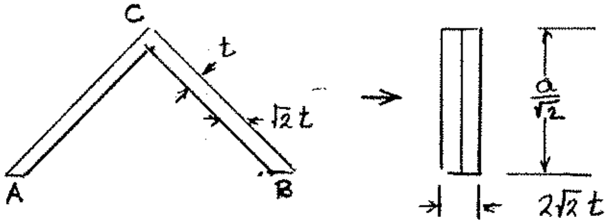
Answer 34.7 mm from point A ◄

Problem 4.192

4.192 The shape shown was formed by bending a thin steel plate. Assuming that the thickness *t* is small compared to the length *a* of a side of the shape, determine the stress (*a*) at *A*, (*b*) at *B*, (*c*) at *C*.



Moment of inertia about centroid



$$I = \frac{1}{12}(2\sqrt{2}t)\left(\frac{a}{\sqrt{2}}\right)^3$$
$$= \frac{1}{12}ta^3$$

Area $\quad A = (2\sqrt{2}t)\left(\frac{a}{\sqrt{2}}\right) = 2at, \qquad c = \frac{a}{2\sqrt{2}}$

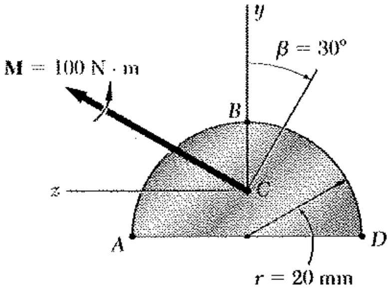
(a) $\quad \sigma_A = \frac{P}{A} - \frac{Pec}{I} = \frac{P}{2at} - \frac{P\left(\frac{a}{2\sqrt{2}}\right)\left(\frac{a}{2\sqrt{2}}\right)}{\frac{1}{12}ta^3} = \qquad \sigma_A = -\frac{P}{2at}$ ◄

(b) $\quad \sigma_B = \frac{P}{A} + \frac{Pec}{I} = \frac{P}{2at} + \frac{P\left(\frac{a}{\sqrt{2}}\right)\left(\frac{a}{\sqrt{2}}\right)}{\frac{1}{12}ta^3} \qquad \sigma_B = \frac{2P}{at}$ ◄

(c) $\quad \sigma_C = \sigma_A \qquad \sigma_C = -\frac{P}{2at}$ ◄

Problem 4.193

4.193 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.



$$I_z = \frac{\pi}{8}r^4 - \left(\frac{\pi}{2}r^2\right)\left(\frac{4r}{3\pi}\right)^2 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)r^4$$

$$= (0.109757)(20)^4 = 17.5611\times10^{-3}\ \mathrm{mm}^4 = 17.5611\times10^{-9}\ \mathrm{m}^4$$

$$I_y = \frac{\pi}{8}r^4 = \frac{\pi(20)^4}{8} = 62.832\times10^{-3}\ \mathrm{mm}^4 = 62.832\times10^{-9}\ \mathrm{m}^4$$

$$y_A = y_D = -\frac{4r}{3\pi} = -\frac{(4)(20)}{3\pi} = -8.4883\ \mathrm{mm}$$

$$y_B = 20 - 8.4883 = 11.5117\ \mathrm{mm}$$

$$z_A = -z_D = 20\ \mathrm{mm} \qquad z_B = 0$$

$$M_z = 100\cos 30^\circ = 86.603\ \mathrm{N\cdot m}$$

$$M_y = 100\sin 30^\circ = 50\ \mathrm{N\cdot m}$$

(a) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(86.603)(-8.4883\times10^{-3})}{17.5611\times10^{-9}} + \frac{(50)(20\times10^{-3})}{62.832\times10^{-9}}$$

$$= 57.8\times10^{6}\ \mathrm{Pa} \qquad \sigma_A = 57.8\ \mathrm{MPa}$$ ◄

(b) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(86.603)(11.5117\times10^{-3})}{17.5611\times10^{-9}} + \frac{(50)(0)}{62.832\times10^{-9}}$$

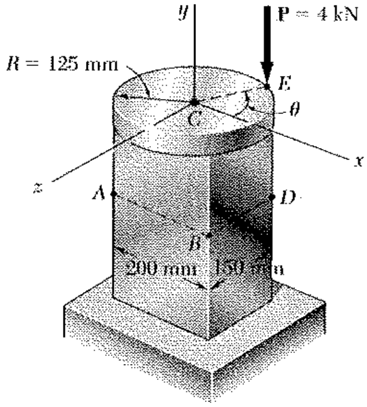
$$= -56.8\times10^{6}\ \mathrm{Pa} \qquad \sigma_B = -56.8\ \mathrm{MPa}$$ ◄

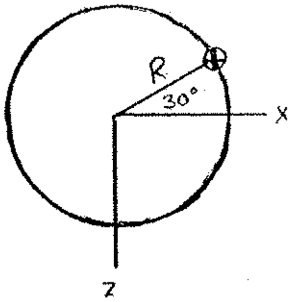
(c) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(86.603)(-8.4883\times10^{-3})}{17.5611\times10^{-9}} + \frac{(50)(-20\times10^{3})}{62.832\times10^{-9}}$$

$$= 25.9\times10^{6}\ \mathrm{Pa} \qquad \sigma_c = 25.9\ \mathrm{MPa}$$ ◄

Problem 4.194

4.194 A rigid circular plate of 125-mm radius is attached to a solid 150 × 200-mm rectangular post, with the center of the plate directly above the center of the post. If a 4-kN force **P** is applied at *E* with $\theta = 30°$, determine (*a*) the stress at point *A*, (*b*) the stress at point *B*, (*c*) the point where the neutral axis intersects line *ABD*.

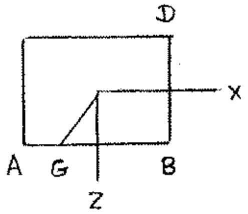




$P = 4 \times 10^3$ N (compression)

$$M_x = -PR \sin 30° = -(4\times10^3)(125\times10^{-3}) \sin 30° = -250 \text{ N}\cdot\text{m}$$

$$M_z = -PR \cos 30° = -(4\times10^3)(125\times10^{-3}) \cos 30° = -433 \text{ N}\cdot\text{m}$$



$$I_x = \frac{1}{12}(200)(150)^3 = 56.25\times10^6 \text{ mm}^4 = 56.25\times10^{-6} \text{ m}^4$$

$$I_z = \frac{1}{12}(150)(200)^3 = 100\times10^6 \text{ mm}^4 = 100\times10^{-6} \text{ m}^4$$

$$-x_A = x_B = 100 \text{ mm}$$

$$z_A = z_B = 75 \text{ mm}$$

$$A = (200)(150) = 30\times10^3 \text{ mm}^2 = 30\times10^{-3} \text{ m}^2$$

(a) $$\sigma_A = -\frac{P}{A} - \frac{M_x z_A}{I_x} + \frac{M_z x_A}{I_z} = -\frac{4\times10^3}{30\times10^{-3}} - \frac{(-250)(75\times10^{-3})}{56.25\times10^{-6}} + \frac{(-433)(-100\times10^{-3})}{100\times10^{-6}}$$

$$= 633\times10^3 \text{ Pa} = 633 \text{ kPa}$$ ◀

(b) $$\sigma_B = -\frac{P}{A} - \frac{M_x z_B}{I_x} + \frac{M_z x_B}{I_z} = -\frac{4\times10^3}{30\times10^{-3}} - \frac{(-250)(75\times10^{-3})}{56.25\times10^{-6}} + \frac{(-433)(100\times10^{-3})}{100\times10^{-6}}$$

$$= -233\times10^3 \text{ Pa} = -233 \text{ kPa}$$ ◀

(c) Let G be the point on AB where neutral axis intersects.

$\sigma_G = 0$ $\quad z_G = 75$ mm $\quad x_G = ?$

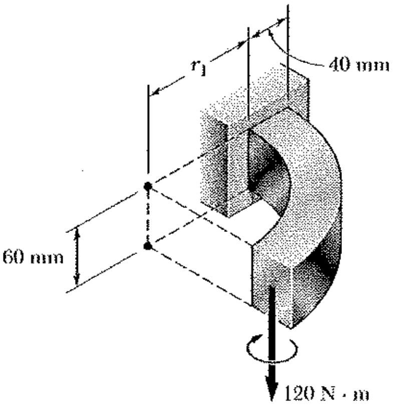
$$\sigma_G = -\frac{P}{A} - \frac{M_x z_G}{I_x} + \frac{M_z x_G}{I_z} = 0$$

$$x_G = \frac{I_z}{M_z}\left\{\frac{P}{A} + \frac{M_x z_G}{I_x}\right] = \frac{100\times10^{-6}}{-433}\left\{\frac{4\times10^3}{30\times10^{-3}} + \frac{(-250)(75\times10^{-3})}{56.25\times10^{-6}}\right\}$$

$$= 46.2\times10^{-3} \text{ m} = 46.2 \text{ mm}$$

Point G lies 146.2 mm from point A. ◀

Problem 4.195



4.195 The curved bar shown has a cross section of 40 × 60 mm and an inner radius $r_1 = 15$ mm. For the loading shown, determine the largest tensile and compressive stresses in the bar.

$h = 40$ mm, $r_1 = 15$ mm, $r_2 = 55$ mm

$A = (60)(40) = 2400 \text{ mm}^2 = 2400 \times 10^{-6} \text{ m}^2$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{40}{\ln \frac{55}{40}} = 30.786 \text{ mm}$$

$\bar{r} = \frac{1}{2}(r_1 + r_2) = 35$ mm

$e = \bar{r} - R = 4.214$ mm $\qquad \sigma = -\dfrac{My}{Aer}$

At $r = 15$ mm $\quad y = 30.786 - 15 = 15.786$ mm

$$\sigma = -\frac{(120)(15.786 \times 10^{-3})}{(2400 \times 10^{-6})(4.214 \times 10^{-3})(15 \times 10^{-3})} = -12.49 \times 10^6 \text{ Pa}$$

$= -12.49$ MPa ◀

(compression)

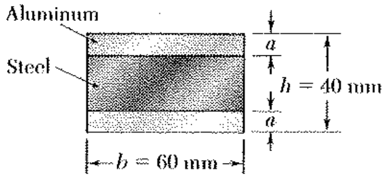
At $r = 55$ mm $\quad y = 30.786 - 55 = -24.214$ mm

$$\sigma = -\frac{(120)(-24.214 \times 10^{-3})}{(2400 \times 10^{-6})(4.214 \times 10^{-3})(55 \times 10^{-3})} = 5.22 \times 10^6 \text{ Pa}$$

$= 5.22$ MPa ◀

(tension)

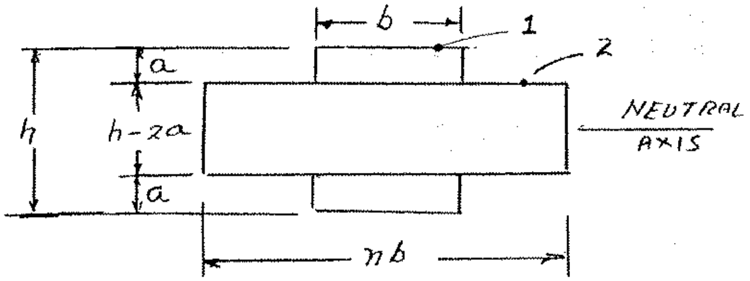
PROBLEM 4.C1

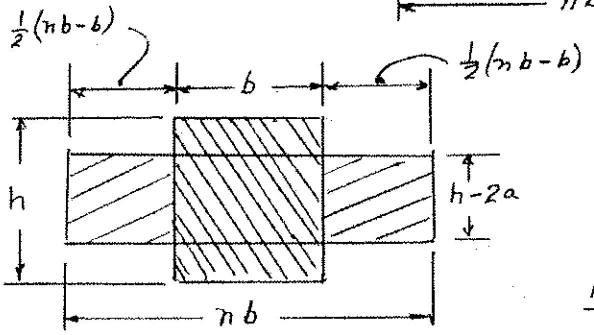


4.C1 Two aluminum strips and a steel strip are to be bonded together to form a composite member of width $b = 60$ mm and depth $h = 40$ mm. The modulus of elasticity is 200 GPa for the steel and 75 GPa for the aluminum. Knowing that $M = 1500$ N · m, write a computer program to calculate the maximum stress in the aluminum and in the steel for values of a from 0 to 20 mm using 2-mm increments. Using appropriate smaller increments, determine (a) the largest stress that can occur in the steel, (b) the corresponding value of a.

SOLUTION

TRANSFORMED SECTION (ALL STEEL) $n = \frac{E_{STEEL}}{E_{ALUM}}$





$$\bar{I} = \frac{1}{12} b h^3 + \frac{1}{12}\left[2\left(\frac{1}{2}\right)(nb - b\right] (h-2a)^3$$

AT POINT 1: $\sigma_{ALUM} = \frac{M\left(\frac{h}{2}\right)}{\bar{I}}$

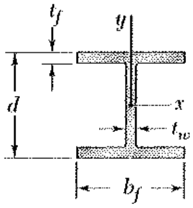
AT POINT 2: $\sigma_{STEEL} = n \frac{M\left(\frac{h}{2} - a\right)}{\bar{I}}$

FOR $a = 0$ TO 20 mm USING 2-mm INTERVALS COMPUTE: n, $\bar{I}$, σ_{ALUM}, σ_{STEEL}.

PROGRAM OUTPUT

```
  b = 60 mm    h = 40 mm     M = 1500 N.m
   Moduli of elasticity:   Steel = 200 GPa  Aluminum = 75 GPa

       a        I          sigma        sigma
                         aluminum      steel
      mm    m^4/10^6        MPa         MPa

   0.000    0.8533      35.156     93.750
   2.000    0.7088      42.325    101.580
   4.000    0.5931      50.585    107.914
   6.000    0.5029      59.650    111.347
   8.000    0.4352      68.934    110.294
  10.000    0.3867      77.586    103.448
  12.000    0.3541      84.714     90.361
  14.000    0.3344      89.713     71.770
  16.000    0.3243      92.516     49.342
  18.000    0.3205      93.594     24.958
  20.000    0.3200      93.750      0.000
  ====================================================
  Find 'a' for max steel stress
      and the corresponding aluminum stress

   6.600    0.4804      62.447    111.572083
   6.610    0.4800      62.494    111.572159
   6.620    0.4797      62.540    111.572113

    Max Steel Stress = 111.6 MPa occurs when a = 6.61 mm
    Corresponding Aluminum stress = 62.5 MPa
```

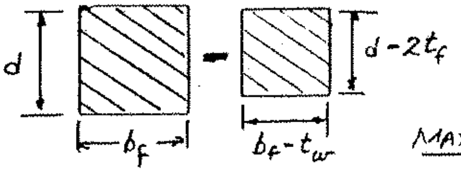
PROBLEM 4.C2



4.C2 A beam of the cross section shown, made of a steel that is assumed to be elastoplastic with a yield strength σ_Y and a modulus of elasticity E, is bent about the x axis. (*a*) Denoting by y_Y the half thickness of the elastic core, write a computer program to calculate the bending moment M and the radius of curvature ρ for values of y_Y from $\frac{1}{2}d$ to $\frac{1}{6}d$ using decrements equal to $\frac{1}{2}t_f$. Neglect the effect of fillets. (*b*) Use this program to solve Prob. 4.190

SOLUTION

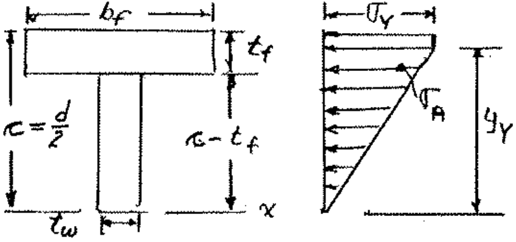
COMPUTE MOMENT OF INERTIA I_x



$$I_x = \frac{1}{12} b_f d^3 - \frac{1}{12}(b_f - t_w)(d-2t_f)^3$$

MAXIMUM ELASTIC MOMENT: $M_Y = \sigma_Y \frac{I_x}{(d/2)}$

FOR YIELDING IN THE FLANGES: (CONSIDER UPPER HALF OF CROSS SECTION) $c = \frac{d}{2}$

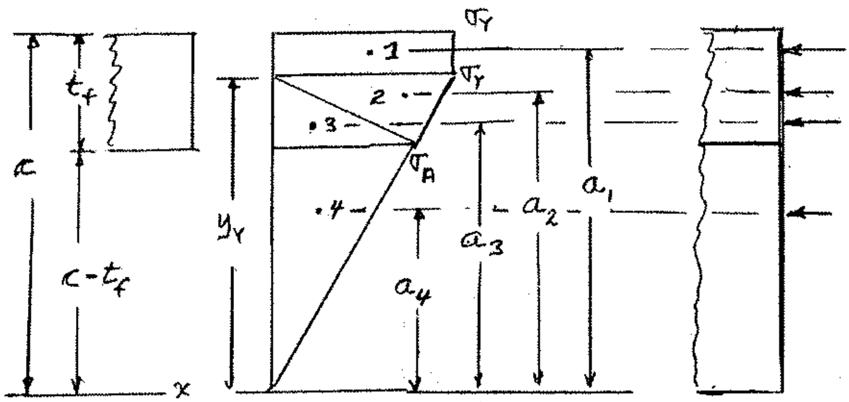


STRESS AT JUNCTION OF WEB AND FLANGE

$$\sigma_A = \frac{(d/2) - t_f}{y_Y}\sigma_Y$$

DETAIL OF STRESS DIAGRAM

RESULTANT FORCES



$R_1 = \sigma_Y[b_f(c - \sigma_Y)]$

$R_2 = \frac{1}{2}\sigma_Y b_f[y_Y - (c - t_f)]$

$R_3 = \frac{1}{2}\sigma_A b_f[y_Y - (c - t_f)]$

$R_3 = \frac{1}{2}\sigma_A t_w (c - t_f)$

$a_1 = \frac{1}{2}(c + y_Y)$

$a_2 = y_Y - \frac{1}{3}[y_Y - (c - t_f)]$

$a_3 = y_Y - \frac{2}{3}[y_Y - (c - t_f)]$

$a_4 = \frac{2}{3}(c - t_f)$

BENDING MOMENT

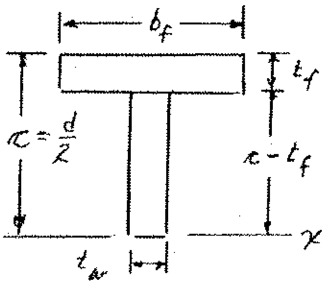
$$M = 2\sum_{n=1}^{4} R_n a_n$$

RADIUS OF CURVATURE

$$y_Y = \varepsilon_Y \rho = \frac{\sigma_Y}{E}\rho \; ; \quad \rho = \frac{y_Y E}{\sigma_Y}$$

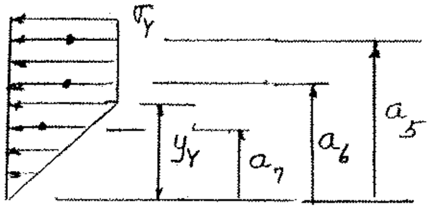
CONTINUED

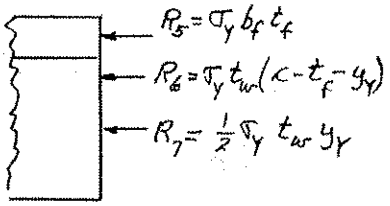
PROBLEM 4.C2 - CONTINUED



FOR YIELDING IN THE WEB (CONSIDER UPPER HALF OF CROSS SECTION)

$c = d/2$





$$a_5 = c - \frac{1}{2}t_f$$

$$a_6 = \frac{1}{2}[y_Y + (c - t_f)]$$

$$a_7 = \frac{2}{3} y_Y$$

BENDING MOMENT $\quad M = 2\sum_{n=5}^{7} R_n a_n$

RADIUS OF CURVATURE $\quad y_Y = \epsilon_y \rho = \frac{\sigma_Y}{E}\rho \qquad \rho = \frac{y_Y E}{\sigma_Y}$

PROGRAM: KEY IN EXPRESSIONS FOR a_n and R_n FOR $n = 1$ TO 7.

FOR $y_Y = c$ TO $(c - t_f)$ AT $-t_f/2$ DECREMENTS
COMPUTE $M = 2\Sigma R_n a_n$ FOR $n = 1$ TO 4 AND $\rho = \frac{y_Y E}{\sigma_Y}$, THEN PRINT

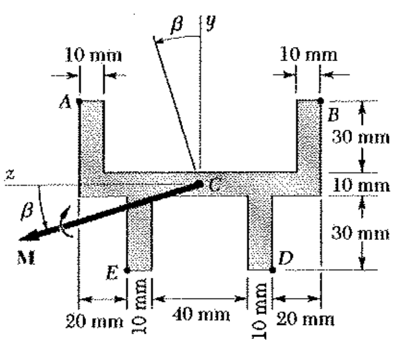
FOR $y_Y = (c - t_w)$ TO $c/3$ AT $-t_f/2$ DECREMENTS
COMPUTE $M = 2\Sigma R_n a_n$ FOR $n = 5$ TO 7. AND $\rho = \frac{y_Y E}{\sigma_Y}$, THEN PRINT

INPUT NUMERICAL VALUES AND RUN PROGRAM

PROGRAM OUTPUT

```
 For a beam of Prob 4.190
Depth d = 140.00 mm                          Width of flange bf = 120.00 mm
Thickness of flange tf =  10.00 mm    Thickness of web tw =  10.00 mm

I = 0.000011600 m to the 4th
Yield strength of Steel sigmaY = 300 MPa
Yield Moment MY = 49.71 kip.in.

        yY(mm)           M(kN.m)     rho(m)
 For yielding still in the flange.
       70.000             49.71        46.67
       65.000             52.59        43.33
       60.000             54.00        40.00
 For yielding in the web
       60.000             54.00        40.00
       55.000             54.58        36.67
       50.000             55.10        33.33
       45.000             55.58        30.00
       40.000             56.00        26.67
       35.000             56.38        23.33
       30.000             56.70        20.00
       25.000             56.97        16.67
```

PROBLEM 4.C3



4.C3 A 900 N · m couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Noting that the centroid of the cross section is located at C and that the y and z axes are principal axes, write a computer program to calculate the stress at A, B, C, and D for values of β from 0 to 180° using 10° increments. (*Given:* $I_y = 2.59 \times 10^6$ mm^4 and $I_z = 0.62 \times 10^6$ mm^4.)

SOLUTION

$(\times 10^{-3})$

INPUT COORDINATES OF A, B, C, D

$z_A = z(1) = 50$ $\quad y_A = y(1) = 35$

$z_B = z(2) = -50$ $\quad y_B = y(2) = 35$

$z_D = z(3) = -30$ $\quad y_D = y(3) = -35$

$z_E = z(4) = 30$ $\quad y_E = y(4) = -35$

COMPONENTS OF M.

$M_y = -M \sin\beta$ $\qquad M_z = M\cos\beta$

Eq. 4.55 page 273: $\quad \sigma(n) = -\dfrac{M_z\, y(n)}{I_z} + \dfrac{M_y\, z(n)}{I_y}$

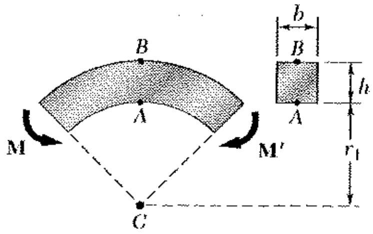
PROGRAM:
```
FOR β = 0 TO 180° USING 10° INCREMENTS.
    FOR n = 1 TO 4 USING UNIT INCREMENTS.
        EVALUATE EQ 4.55 AND PRINT STRESSES
    RETURN
RETURN
```

PROGRAM OUTPUT

```
Moment of couple M = 900 N.m
Moments of inertia: Iy = 2.59X10^-6 m^4      Iz  = 0.62X10^-6 m^4.

     Coordinates of points A, B, D, and E  (X10^-3)
   Point A:  z(1) = 50:      y(1) =  35
   Point B:  z(2) =-50:      y(2) =  35
   Point D:  z(3) =-30:      y(3) = -35
   Point E:  z(4) = 30:      y(4) = -35
           - - - Stress at Points - - -
```

beta deg	A MPa	B MPa	D MPa	E MPa
0	-52.161	-52.161	52.161	52.161
40	-51.340	-28.573	45.652	34.268
90	-17.713	17.713	8.853	-8.853
130	19.961	47.093	-26.746	-40.308
180	52.161	52.161	-52.161	-52.161

PROBLEM 4.C4



4.C4 Couples of moment $M = 2$ kN · m are applied as shown to a curved bar having a rectangular cross section with $h = 100$ mm and $b = 25$ mm. Write a computer program and use it to calculate the stresses at points A and B for values of the ratio r_1/h from 10 to 1 using decrements of 1, and from 1 to 0.1 using decrements of 0.1. Using appropriate smaller increments, determine the ratio r_1/h for which the maximum stress in the curved bar is 50 percent larger than the maximum stress in a strainght bar of the same cross section.

SOLUTION INPUT: $h = 100$ mm, $b = 25$ mm, $M = 2$ kN·m

FOR STRAIGHT BAR: $\sigma_{STRAIGHT} = \frac{M}{S} = \frac{6M}{h^2 b} = 48$ MPa

FOLLOWING NOTATION OF SEC. 4.15, KEY IN THE FOLLOWING:

$r_2 = h + r_1$; $R = h/\ln(r_2 - r_1)$; $\bar{r} = r_1 + r_2$: $e = \bar{r} - R$; $A = bh = 2500$ (I)

STRESSES: $\sigma_A = \sigma_1 = M(r_1 - R)/(Aer_1)$ $\qquad \sigma_B = \sigma_2 = M(r_2 - R)/(Aer_2)$ (II)

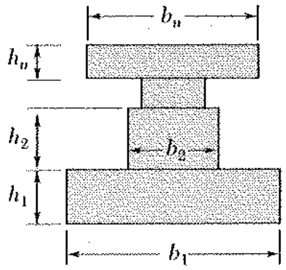
SINCE $h = 100$ mm, FOR $r_1/h = 10$, $r_1 = 1000$ mm. ALSO $r_1/h = 10$, $r_1 = 100$

```
PROGRAM:  FOR r1 = 1000 TO 100 AT -100 DECREMENTS
              USING EQUATIONS OF LINES I AND II EVALUATE r2, R, r̄, e, σ1 AND σ2
              ALSO EVALUATE:  ratio = σ1/σSTRAIGHT
          RETURN AND REPEAT FOR r1 = 100 TO 10 AT -10 DECREMENT
```

PROGRAM OUTPUT

```
     M = Bending Moment = 2.  kN.m     h = 100.000 in.    A = 2500.00 mm^2
     Stress in straight beam  =  48.00 MPa

      r1      rbar     R        e        sigma1     sigma2      r1/h      ratio
      mm       mm      mm       mm         MPa        MPa        -          -
     1000     1050    1049    0.794      -49.57     46.51     10.000    -1.033
      900      950     949    0.878      -49.74     46.36      9.000    -1.036
      800      850     849    0.981      -49.95     46.18      8.000    -1.041
      700      750     749    1.112      -50.22     45.95      7.000    -1.046
      600      650     649    1.284      -50.59     45.64      6.000    -1.054
      500      550     548    1.518      -51.08     45.24      5.000    -1.064
      400      450     448    1.858      -51.82     44.66      4.000    -1.080
      300      350     348    2.394      -53.03     43.77      3.000    -1.105
      200      250     247    3.370      -55.35     42.24      2.000    -1.153
      100      150     144    5.730      -61.80     38.90      1.000    -1.288
          ==============================================
      100      150     144    5.730      -61.80     38.90      1.000    -1.288
       90      140     134    6.170      -63.15     38.33      0.900    -1.316
       80      130     123    6.685      -64.80     37.69      0.800    -1.350
       70      120     113    7.299      -66.86     36.94      0.700    -1.393
       60      110     102    8.045      -69.53     36.07      0.600    -1.449
       50      100      91    8.976      -73.13     35.04      0.500    -1.523
       40       90      80   10.176      -78.27     33.79      0.400    -1.631
       30       80      68   11.803      -86.30     32.22      0.300    -1.798
       20       70      56   14.189     -100.95     30.16      0.200    -2.103
       10       60      42   18.297     -138.62     27.15      0.100    -2.888
          =======================================================
    Find r1/h for (sigma max)/(sigma straight)  = 1.5
       52.70  103       94    8.703     -72.036     35.34      0.527    -1.501
       52.80  103       94    8.693     -71.998     35.35      0.528    -1.500
       52.90  103       94    8.683     -71.959     35.36      0.529    -1.499
   Ratio of stresses is 1.5 for r1 = 52.8 mm or r1/h = 0.529

       [ Note: The desired ratio r1/h is valid for any beam
               having a rectangular cross section. ]
```

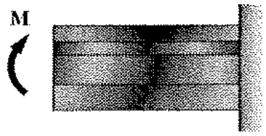
PROBLEM 4.C5

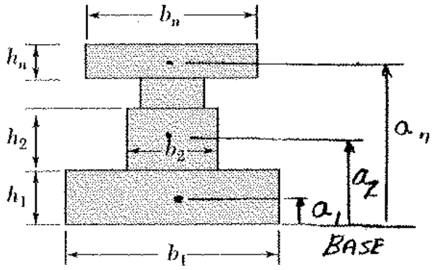


4.C5 The couple **M** is applied to a beam of the cross section shown. (*a*) Write a computer program that, for loads expressed in either SI or U.S. customary units, can be used to calculate the maximum tensile and compressive stresses in the beam. (*b*) Use this program to solve Probs. 4.7, 4.8, and 4.9.

SOLUTION

INPUT: BENDING MOMENT M





FOR $n = 1$ TO n: ENTER b_n AND h_n

$\Delta AREA = b_n h_n$ (PRINT)

$a_n = a_{n-1} + (h_{n-1})/2 + h_n/2$

[MOMENT OF RECTANGLE ABOUT BASE]

$\Delta m = (\Delta AREA)\, a_n$

[FOR WHOLE CROSS SECTION]

$m = m + \Delta m$; $AREA = AREA + \Delta AREA$

LOCATION OF CENTROID ABOVE BASE

$\bar{y} = m/AREA$ (PRINT)

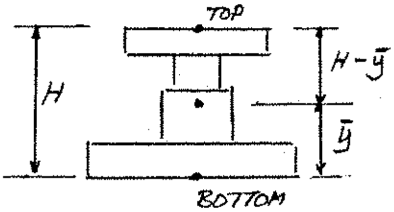
MOMENT OF INERTIA ABOUT HORIZONTAL CENTROIDAL AXIS

FOR $n = 1$ TO n: $a_n = a_{n-1} + (h_{n-1})/2 + h_n/2$

$\Delta I = b_n h_n^3/12 + (b_n h_n)(\bar{y} - a_n)^2$

$I = I + \Delta I$ (PRINT)

COMPUTATION OF STRESSES



TOTAL HEIGHT: FOR $n = 1$ TO n

$H = H + h_n$

STRESS AT TOP

$$M_{TOP} = -M\,\frac{H-\bar{y}}{I}$$ (PRINT)

STRESS AT BOTTOM

$$M_{BOTTOM} = M\,\frac{\bar{y}}{I}$$ (PRINT)

SEE NEXT PAGE FOR PRINTOUTS

CONTINUED

PROBLEM 4.C5 - CONTINUED

Problem 4.7

```
Summary of Cross Section Dimensions
      Width (m)       Height (m)
        0.225            0.05
        0.075            0.15

Bending Moment = 60,000 Nm
Centroid is 0.075 m above lower edge
Centroidal Moment of Inertia is   79.  x 10^-6m^4

Stress at top of beam    =  -94.103 MPa
Stress at bottom of beam =   56.462 MPa
```

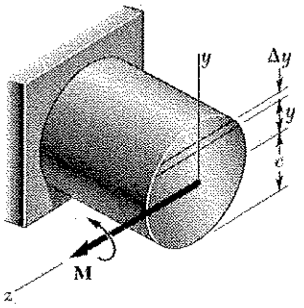
Problem 4.8

```
Summary of Cross Section Dimensions
      Width (m)       Height (m)
        0.1             0.025
        0.025           0.15
        0.2             0.025

Bending Moment = 60000 Nm
Centroid is   0.1194 m above lower edge
Centroidal Moment of Inertia is   60.59 x 10^-6m^4

Stress at top of beam    =  -79.815 MPa
Stress at bottom of beam =  118.237 MPa
```

PROBLEM 4.9

```
Summary of Cross Section Dimensions
     Width (mm)        Height (mm)
       50                10
       20                50

Bending Moment =    1500.0000 N.m
Centroid is   25.000 mm above lower edge
Centroidal Moment of Inertia is      512500 mm^4

Stress at top of beam    = -102.439 MPa
Stress at bottom of beam =   73.171 MPa
```

PROBLEM 4.C6



4.C6 A solid rod of radius $c = 30$ mm is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 290$ MPa. The rod is subjected to a couple of moment M that increases from zero to the maximum elastic moment M_Y and then to the plastic moment M_p. Denoting by y_Y the half thickness of the elastic core, write a computer program and use it to calculate the bending moment M and the radius of curvature ρ for values of y_Y from 30 mm to 0 using 5-mm decrements. (*Hint:* Divide the cross section into 80 horizontal elements of 1-mm height.)

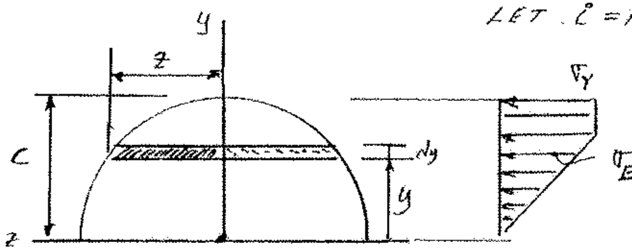
SOLUTION $M_Y = \sigma_Y \frac{\pi}{4} c^3 = (290 \times 10^6) \frac{\pi}{4} (0.03)^3 = 615 \text{ MPa}$

$$M_p = \sigma_Y \frac{4}{3} c^3 = (290 \times 10^6) \frac{4}{3} (0.03)^3 = 104.4 \text{ kNm}$$

CONSIDER TOP HALF OF ROD

LET i = NUMBER OF ELEMENTS IN TOP HALF

Δh = HEIGTH OF EACH ELEMENT: $\Delta h = \frac{c}{i}$



σ_E = STRESS ON ELEMENT

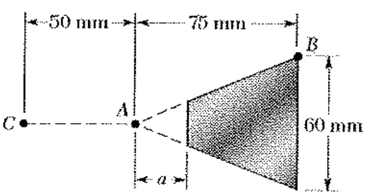
```
FOR n = 0 TO i-1 STEP 1
    $y = n(\Delta h)$
    $z = [c^2 - \{(n+0.5)\Delta h\}^2]$          ← z AT MIDHEIGHT OF ELEMENT
    IF $y \geq y_Y$ GO TO 100
        $\sigma_E = \sigma_Y \frac{(n+0.5)\Delta h}{\sigma_y}$          ← STRESS IN ELASTIC CORE
    GOTO 200
100     $\sigma_E = \sigma_Y$          ← STRESS IN PLASTIC ZONE
200 $\Delta AREA = 2z(\Delta h)$
    $\Delta FORCE = \sigma_E (\Delta AREA)$
    $\Delta MOMENT = \Delta FORCE (n+.5)\Delta h$
    $M = M + \Delta MOMENT$
    $\rho = y_Y E / \sigma_Y$
    PRINT $y_Y$, M, AND $\rho$.
NEXT
```

REPEAT FOR $y_Y = 30$mm TO $y_Y = 0$ AT 5mm DECREMENTS

PROGRAM OUTPUT

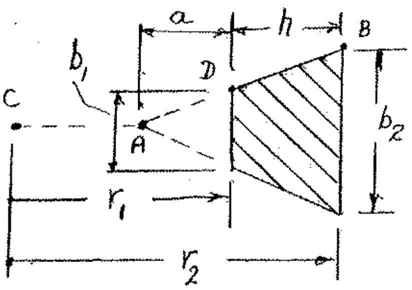
```
Radius of rod = 0.03 m
Yield point of steel =  290 MPa
Yield moment = 615 MPa              Plastic moment = 104.4 KNm
Number of elements in half of the rod =  40
                    YY (mm)        P (mm x 10³)
                     30              20.690
                     25              17.241
                     20              13.793
                     15              10.345
                     10               6.897
                      5               3.448
                      0                0
```

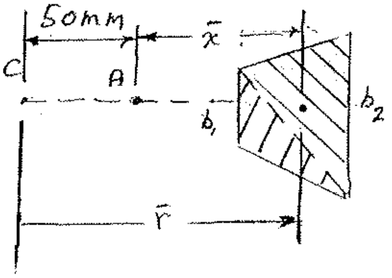
PROBLEM 4.C7



4.C7 The machine element of Prob. 4.178 is to be redesigned by removing part of the triangular cross section. It is believed that the removal of a small triangular area of width *a* will lower the maximum stress in the element. In order to verify this design concept, write a computer program to calculate the maximum stress in the element for values of *a* from 0 to 25 mm using 2.5-mm increments. Using appropriate smaller increments, determine the distance *a* for which the maximum stress is as small as possible and the corresponding value of the maximum stress.

SOLUTION SEE FIG 4.79 PAGE 289





$M = 560 Nm \quad r_2 = 125 mm \quad b_2 = 62 mm$

FOR a = 0 TO 1.0 AT 0.1 INTERVALS

$h = 3 - a$

$r_1 = 2 + a$

$b_1 = b_2 (a/(h+a))$

$AREA = (b_1 + b_2)(h/2)$

$\bar{x} = a + \left[\frac{1}{2} b_1 h (h/3) + \frac{1}{2} b_2 h (2h/3)\right] / AREA$

$\bar{r} = r_2 - (h - \bar{x})$

$$R = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(b_1 r_2 - b_2 r_1) \ln \frac{r_2}{r_1} - h(b_1 - b_2)}$$

$e = \bar{r} - R$

$\sigma_D = M(r_1 - R) / [AREA(e)(r_1)]$

$\sigma_B = M(r_2 - R) / [AREA(e)(r_2)]$

PRINT AND RETURN

PROGRAM OUTPUT

```
  a        R        sigmaD     sigmaB     b1       rbar      e
  mm       mm       MPa        MPa        mm       mm        mm.
  0      97.917   -58.656    14.489      0      101.600   3.683
  5      98.273   -50.127    14.955    4.318    101.854   3.556
 12.5    99.771   -45.288    16.990   10.668    102.870   3.023
 20     102.057   -45.575    20.583   17.018    104.394   2.388
 25     103.861   -48.022    23.782   21.082    105.918   1.981.
----------------------------------------------------------------------
Determination of the maximum compressive stress that is as small as possible
  a        R        sigmaD     sigmaB     b1       rbar      e
  mm       mm       MPa        MPa        mm       mm        mm.
 15.5   100.669  -44.9001    18.2200    1.321    10.338    0.277
 15.625 100.660  -44.8992    18.2766    1.321    10.338    0.277
 15.75  100.686  -44.8994    18.3345    1.321    10.338    0.277.
```

Answer: When a = 15.625 mm the compressive stress is 44.9 MPa